

1. Find the limits.

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{x^2+1} - \sqrt{1-x^2}}{x^2}$

(b) $\lim_{x \rightarrow 2^-} \frac{|x^2-3x+2| \cos(2x)}{x^2-3x+2}$

(c) $\lim_{x \rightarrow 0} \frac{\sin(x^2-x)}{\tan(3x)}$

(d) $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$

2. Find the derivatives of the functions. Do not simplify your answers.

(a) $f(x) = x^2 \sin(x^2)$

(b) $g(x) = \cos(\sec(x-1))$

(c) $h(x) = \frac{x^2+2}{\sqrt{x-x^3}}$

3. Consider the function

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & x < 2 \\ ax^2 + bx + 3 & 2 \leq x < 3 \\ 2x - a - b & x \geq 3 \end{cases}$$

Find values of a and b such that $f(x)$ is continuous everywhere.

4. Suppose that $f(x) = \sqrt{x-1}$. Let $\epsilon = 1$. What is the maximum $\delta > 0$ such that $0 < |x-5| < \delta$ implies that $|f(x) - 2| < \epsilon$?
5. Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{x^2}$. (Your answer should involve a computation using a limit.)
6. Prove that the equation $x \cos x = x^2 - 1$ has a root in $[0, \pi]$. Make sure you explain why the hypotheses of any theorems you apply are justified!
7. Find the tangent line to the curve defined by the equation $y \sin(2x) = x \cos(2y)$ at $(\frac{\pi}{2}, \frac{\pi}{4})$.
8. Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the cars increasing two hours later?
9. A 5 ft tall woman walks toward a 20 ft lamppost at a rate of 4 ft/s. How fast is the length of her shadow decreasing when she is 6 ft from the lamppost?
10. A ball is thrown into the air from atop a cliff on the moon. Its height above the ground, in feet, is given by $s(t) = 100 + 20t - 26t^2$ ft.
- (a) At what time does the ball hit the ground?
- (b) What is the ball's position when its velocity is -58 ft/s?
- (c) What is the maximum height the ball reaches? You do not need to simplify your answer.