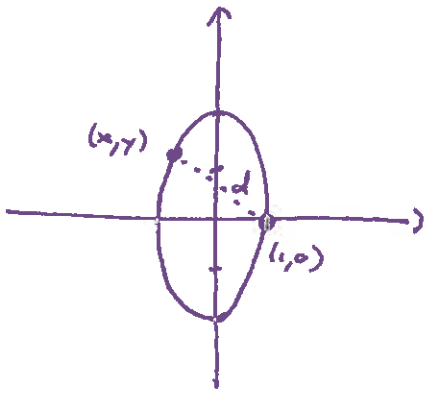
**Name:** _____

1. Class notes for this week: This week we have covered Sections 3.7 and 3.8. Next week we will cover Section 3.9, Section 4.1, and Appendix E, which covers background material for the notation and computations in Section 4.1.

2. (3 points) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.




Minimize distance $d = \sqrt{(x-1)^2 + y^2}$

Equivalently square of distance $D = (x-1)^2 + y^2$

Know $4x^2 + y^2 = 4 \rightsquigarrow y^2 = 4 - 4x^2$

$D(x) = (x-1)^2 + 4 - 4x^2$

$D'(x) = 2(x-1) - 8x$

$0 = 2x - 2 - 8x$

$2 = -6x$

$\frac{-1}{3} = x$

$y^2 = 4 - 4x^2$

$y^2 = 4 - \frac{4}{9}$

$y^2 = \frac{32}{9}$

$y = \pm\frac{4\sqrt{2}}{3}$

So our candidate points are $\left(\frac{-1}{3}, \pm\frac{4\sqrt{2}}{3}\right)$. Let's double-check: $x$ is in the closed interval $[-1, 1]$. At $x = 1$, $D = 0$. At $x = -1$, $D = (2)^2 = 4$. And at the critical number $x = \frac{-1}{3}$, $D = \left(\frac{-1}{3} - 1\right)^2 + 4 - 4\left(\frac{-1}{3}\right)^2$

$= \left(\frac{4}{3}\right)^2 + 4 - \frac{4}{9}$

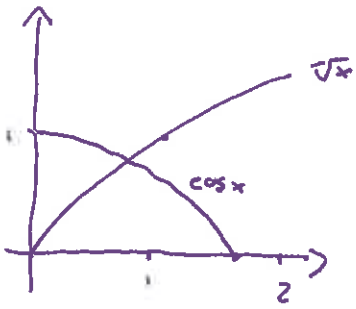
$= \frac{16}{9} + 4 - \frac{4}{9}$

$= \frac{48}{9}$ ✓ Maximum.

Question 3 is on the back

3. (2 points) Use Newton's method to find the roots of $\cos x = \sqrt{x}$ correct to four decimal places.

We want a root of $F(x) = \cos x - \sqrt{x}$.




Start w/ $x_1 = .5$ as a first approximation.

$$F'(x) = -\sin x - \frac{1}{2\sqrt{x}}$$

In general $x_n = x_{n-1} - \dfrac{F(x_{n-1})}{F'(x_{n-1})} = x_{n-1} + \dfrac{\cos(x_{n-1}) - \sqrt{x_{n-1}}}{\sin(x_{n-1}) + \frac{1}{2\sqrt{x_{n-1}}}}$

Putting this into a calculator, we see

$x_2 = .64368$

$x_3 = .64171$

$x_4 = .64171$

$\rightsquigarrow$ we conclude the root is $x \approx .6417$.