

Name: _____

Clear your desk of everything excepts pens, pencils and erasers. If you have a question raise your hand and I will come to you.

1. (2 points) Multiple Choice. No work needed. No partial credit available. Compute the following limit.

$$\lim_{x \rightarrow \infty} \sqrt{9x^2 + x} - 3x$$

A. ∞

B. $\frac{1}{12}$

C. $\frac{1}{6}$

D. $-\infty$

E. 1

Extra Work Space.

$$\textcircled{1} \lim_{x \rightarrow \infty} \sqrt{9x^2 + x} - 3x = \lim_{x \rightarrow \infty} \left(\sqrt{9x^2 + x} + 3x \right) \left(\frac{\sqrt{9x^2 + x} - 3x}{\sqrt{9x^2 + x} + 3x} \right) = \lim_{x \rightarrow \infty} \frac{9x^2 + x - 9x^2}{\sqrt{9x^2 + x} + 3x} = \lim_{x \rightarrow \infty} \frac{x}{\sqrt{9x^2 + x} + 3x}$$

$$\rightarrow \lim_{x \rightarrow \infty} \frac{1}{\sqrt{9 + \frac{1}{x}} + 3} = \frac{1}{\sqrt{9} + 3} = \frac{1}{6}$$

3. (3 points) Draw a graph of the curve $y = (x - 3)\sqrt{x}$. Make sure to clearly identify any intercepts, asymptotes, intervals of increase and decrease, local extrema, and the concavity of the curve.

Domain $x \geq 0$

Intercepts y-intercept $y=0$
 x-intercepts $x=3, 0$

Symmetry None

Asymptotes None

Local Maxima/Minima

$(0, 0)$ is a local max

$(1, -2)$ is a local min

Concavity

$$f''(x) = \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{x}} + (x-3)\left(\frac{-1}{4x^{3/2}}\right)$$

$$= \frac{1}{\sqrt{x}} + (x-3)\left(\frac{-1}{4x^{3/2}}\right)$$

$$0 = \frac{1}{\sqrt{x}} + (x-3)\left(\frac{-1}{4x^{3/2}}\right)$$

$$0 = 4x + (x-3)(-1)$$

$$0 = 4x - x + 3$$

$$0 = 3x + 3$$

$$x = -1$$

~~X~~
 Not in domain

Positive everywhere
 \Rightarrow concave up everywhere

Increasing/Decreasing

$$f'(x) = \sqrt{x} + (x-3)\frac{1}{2\sqrt{x}} \quad \left. \vphantom{f'(x)} \right\} \text{DNE at } x=0$$

$$0 = \sqrt{x} + (x-3)\frac{1}{2\sqrt{x}}$$

$$0 = 2x + x - 3$$

$$3 = 3x$$

$$1 = x$$

$x=0, 1$ are critical numbers

Interval	Sign of $f'(x)$	
$(0, 1)$	-	Decreasing
$(1, \infty)$	+	Increasing

