Name: ______________________

1. Class notes for this week: This week we have covered Sections 3.1, 3.2, and 3.3. Next week we will cover Sections 3.4 and 3.5.

2. In this question, we will show the equation $2x + \cos x = 0$ has exactly one real root.
   (a) (1 point) Let $f(x) = 2x + \cos x$. Use the Intermediate Value Theorem to show there is some c such that $f(c) = 0$ in $(-\frac{\pi}{2}, 0)$. Be sure to check all hypotheses of the theorem.
   (b) (1 point) Suppose there is another number d such that $f(d) = 0$. Use Rolles' Theorem to show that there is some e between d and c such that $f'(e) = 0$. Be sure to check all hypotheses of the theorem.
   (c) (1 point) Show that $f'(x)$ is never zero, and use this to conclude that such a d cannot exist. Therefore c is the only root of $2x + \cos x = 0$.

(a) Note that $F(x)$ is continuous everywhere. We see $F(0) = 2(0)+1 = 1$ and $F(-\frac{\pi}{2}) = 2(-\frac{\pi}{2}) + \cos(-\frac{\pi}{2}) = -\pi$. So since $-\pi < 0 < 1$, there must be a c in $(-\frac{\pi}{2}, 0)$ such that $F(c) = 0$, by IVT

(b) Note F is continuous on $[d, c]$ (or $[c, d]$ as the case may be) and F is differentiable on (d, c) (or (c, d)). So since $F(c) = F(d)$, by Rolles' Theorem there is an e between d and c such that $F'(e) = 0$.

(c) $F'(x) = 2 - \sin x \geq 1$ since $-1 \leq \sin x \leq 1$. So there is no e such that $F'(e) = 0$, implying that there is no d which is a second root of $F(x)$. Ergo c is the only root.

Question 3 is on the back

3. (a) (1 point) Let $g(x) = (x+1)^5 - 5x - 2$. Find
   - The intervals on which g is increasing and decreasing.
   - The local maximum and minimum values of g.
   - The intervals on which g is concave up and concave down.
   - The inflection points of g.

   (b) (1 point) Use your answers to the first part of this problem to sketch a graph of $g(x)$.

(a) $g(x) = (x+1)^5 - 5x - 2$

$g'(x) = 5(x+1)^4 - 5$

$0 = 5(x+1)^4 - 5$

$5 = 5(x+1)^4$

$1 = (x+1)^4$

$\underbrace{x = 0 \quad x = -2}_{\text{Critical numbers of } g}$

$g''(x) = 20(x+1)^3$

$0 = 20(x+1)^3$

$x = -1$

g is increasing on $(-\infty, -2)$ and $(0, \infty)$

g is decreasing on $(-2, 0)$

Local max at $f(-2) = 7$

Local min at $F(0) = -1$

g is concave up on $(-1, \infty)$

g is concave down on $(-\infty, -1)$

g has a point of inflection at $x = -1$.