Name: ____________________

1. Class notes for this week: This week we have covered Sections 1.6 and 1.7. Next week we will cover Sections 1.8, 2.1, and 2.2. Do not forget to do the initial class survey, which is due by September 14.

2. (2 points) Find the right-hand limit

$$\lim_{x\to\frac{\pi}{6}^+} \frac{|x-\frac{\pi}{6}|\sin x}{x-\frac{\pi}{6}}.$$

Show your work, being careful to justify each step in your computation.

Notice that $\lim_{x\to\frac{\pi}{6}^+} \sin x = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ by continuity of $\sin x$.

Moreover, we see $\left|x-\frac{\pi}{6}\right| = \begin{cases} x-\frac{\pi}{6} & x \geq \frac{\pi}{6} \\ \frac{\pi}{6}-x & x < \frac{\pi}{6} \end{cases}$, so

$\lim_{x\to\frac{\pi}{6}^+} \frac{|x-\pi/6|}{x-\frac{\pi}{6}} = \lim_{x\to\frac{\pi}{6}^+} \frac{x-\frac{\pi}{6}}{x-\frac{\pi}{6}} = \lim_{x\to\frac{\pi}{6}^+} 1 = 1$. Ergo by the limit laws for multiplication, $\lim_{x\to\frac{\pi}{6}^+} \frac{|x-\frac{\pi}{6}|\sin x}{x-\frac{\pi}{6}} = 1\left(\frac{1}{2}\right) = \frac{1}{2}$.

Question 3 is on the back

3. (a) (1 point) Follow the outline below to give a formal proof that $\lim_{x\to 0} x^3 = 0$.
"Suppose $\epsilon > 0$ is a very small number. I want to show that, by taking $0 < |x - 0| < \delta$, for some $\delta > 0$, I can always ensure that $|x^3 - 0| < \epsilon$. But if I take $\delta =$ ____, then I see that whenever $|x| = |x - 0| < \delta$, I have that $|x^3 - 0| < \epsilon$. Therefore I conclude that $\lim_{x\to 0} x^3 = 0$.

(b) (2 points) Write out a similar proof that $\lim_{x\to 2} 2x + 1 = 5$.

(a) Want $|x^3 - 0| < \varepsilon$

$\Leftrightarrow |x^3| < \varepsilon$

$\Leftrightarrow -\varepsilon < x^3 < \varepsilon$

$\Leftrightarrow -\varepsilon^{1/3} < x < \varepsilon^{1/3}$

$\Leftrightarrow |x| < \varepsilon^{1/3}$

$\Leftrightarrow |x - 0| < \varepsilon^{1/3} = \delta$

We can take $\delta = \varepsilon^{1/3}$ in the outline above.

(b) First we find δ for arbitrary ε:

<u>Want</u> $|(2x+1) - 5| < \varepsilon$

$|2x - 4| < \varepsilon$

$|x - 2| < \frac{\varepsilon}{2} = \delta$

Now we write the proof:

Suppose $\varepsilon > 0$ is a very small number. I want to show that, by taking $0 < |x-2| < \delta$, I can always ensure $|(2x+1) - 5| < \varepsilon$. But if I take $\delta = \frac{\varepsilon}{2}$, then I see whenever $|x - 2| < \delta$, I have that $|(2x+1) - 5| < \varepsilon$. Therefore $\lim_{x \to 2} (2x+1) = 5$.