Name: ____________________

1. Class notes for this week: This week we have covered Section 3.9, Section 4.1, and Appendix E. Next week we will cover Sections 4.2 and 4.3, and begin review for Exam 2.

2. Do not forget that Exam 2 is Monday, November 21, 7:45-9:15 p.m. We are in Wells A126 for the exam (same room as last time). The exam covers Sections 2.9-3.5,3.7-4.3. It is, as previously, a very good idea to do the exams from previous years which are posted on the course webpage.

3. (2 points) Suppose you know that a particle is traveling with acceleration $a(t) = 10\sin t + 3\cos t$. Furthermore, you know that at time $t = 0$ its position is $s(0) = 0$ and at time $t = 2\pi$ its position is $s(2\pi) = 12$. Determine the position function $s(t)$ of the particle.

$a(t) = 10\sin t + 3\cos t$

$v(t) = -10\cos t + 3\sin t + C_1$

$s(t) = -10\sin t - 3\cos t + C_1 t + C_2$

$0 = s(0) = -10(0) - 3(1) + C_1(0) + C_2$

$0 = 0 - 3 + 0 + C_2$

$3 = C_2$

$12 = s(2\pi) = -10(0) - 3(1) + C_1(2\pi) + 3$

$12 = -3 + 2\pi C_1 + 3$

$\frac{6}{\pi} = C_1$

$$\boxed{s(t) = -10\sin t - 3\cos t + \frac{6}{\pi}t + 3}$$

Question 4 is on the back

4. Let us find the area under the curve $f(x) = x^3$ from $x = 0$ to $x = 1$.

   (a) (1 point) Suppose we divide the interval $[0, 1]$ into n subintervals and use the right-hand endpoints of the intervals as sample points. What is Δx? What is each sample point x_i?

   (b) (1 point) Write an expression for the right-hand sum R_n in sigma notation, and use the summation rules to write this sum as an expression in n.

   (c) (1 point) Find the limit of the sum R_n as $n \to \infty$. What do you conclude the area under the curve is?

ⓐ $\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$

$x_i = a + \frac{i(b-a)}{n} = \frac{i}{n}$

ⓑ $R_n = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3 = \frac{1}{n^4}\sum_{i=1}^{n} i^3 = \frac{1}{n^4}\left[\frac{n(n+1)}{2}\right]^2$

ⓒ $\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{n^2(n+1)^2}{4n^4} = \frac{1}{4}$. So $\int_0^1 x^3\,dx = \frac{1}{4}$.