2010-06-14

Calculus II : Practice final (Calculators and textbooks are **NOT** allowed during the exam.)

1. Evaluate the integral

$$\int \frac{3w-1}{w+2}\,dw.$$

2. Find the value of p for which the series is convergent

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^p}.$$

3. Find the surface area of the surface obtained by rotating

$$y = \sqrt{7-x^2}, \qquad 0 \le x \le 1$$

about the x-axis.

4.

(a) Sketch the curve

$$r = \cos 2\theta.$$

(b) Find the points on the curve $r = \cos 2\theta$ where the tangent line is horizontal or vertical.

(c) Find all points of intersection of the curve $r = \cos 2\theta$ and $r = \frac{1}{2}$.

(d) Find the area enclosed by one loop of the curve $r = \cos 2\theta$.

5. Let $x = e^t + e^{-t}$ and $y = 1 - 2t$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) For $0 \le t \le 3$, find the exact length of the curve.

6. Determine whether the series is conditionally convergent, absolutely convergent, or divergent. Explain why.

(a)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$$

(b)

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{2^{\frac{1}{n}}}$$

(c)

$$\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{4n}}$$

7. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?

(a)

$$\sum_{n=0}^{\infty} c_n$$

(b)

$$\sum_{n=0}^{\infty} c_n 8^n$$

(c)

$$\sum_{n=0}^{\infty} c_n (-3)^n$$

8. Find the radius of convergence and interval of convergence of the series

$$f(x) = \sum_{n=1}^{\infty} \frac{2^n (x-1)^n}{\sqrt{n}}.$$

9.

(a) Find the radius of convergence and interval of convergence of the seires

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

(b) In fact,

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$$

for x in the interval of convergence obtained in (a). Evaluate the integral

$$\int e^{-x^2}\, dx.$$

10. Use differentiation to find a power series representation for

$$f(x) = \frac{1}{(1+x)^2}.$$

What is the radius of convergence?

Trigonometric identities

$$\sin^2 x + \cos^2 x = 1$$
$$\sec^2 x = 1 + \tan^2 x$$
$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$
$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$
$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

Integration formulas Constants of integration have been omitted

$$\int \frac{1}{x^n}\,dx = \begin{cases} \frac{1}{(-n+1)x^{n-1}}, & \text{if } n \neq 1 \\ \ln|x|, & \text{if } n = 1 \end{cases}$$
$$\int e^x\,dx = e^x$$
$$\int \sin x\,dx = -\cos x$$
$$\int \cos x\,dx = \sin x$$
$$\int \sec^2 x\,dx = \tan x$$
$$\int \sec x \tan x\,dx = \sec x$$
$$\int \sec x\,dx = \ln|\sec x + \tan x|$$
$$\int \tan x\,dx = \ln|\sec x|$$