## S1102.D Summer 2011

## Calculus II

## Midterm

Instructions: You have 90 minutes to complete the exam. There are seven problems, worth a total of 120 points. Calculators and textbooks are not allowed. Provide the answers in the simplest possible form that does not require calculator use. (E.g. expressions like $\sqrt{13}$ are fine.) Show all of your work: if you only give the answer you will receive no credit, but conversely, partial credit will be given for partial solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: $\qquad$

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 10 |  |
| 3 | 25 |  |
| 4 | 15 |  |
| 5 | 15 |  |
| 6 | 15 |  |
| 7 | 0 |  |
| Total: | 100 |  |

## Problem 1.

Evaluate the following integrals.
(a) [5pts.]

$$
\int x^{2} 2^{x} d x
$$

(b) [5pts.]

$$
\int \sin ^{3}(\theta) \cos ^{2}(\theta) d \theta
$$

(c) [10pts.]

$$
\int \frac{1}{\left(u^{2}+6 u+25\right)^{\frac{3}{2}}} d u
$$

Problem 2. 10pts.
Evaluate the following integral.

$$
\int \frac{3 x+3}{x^{3}-1} d x
$$

Note: $x^{3}-1=(x-1)\left(x^{2}+x+1\right)$.

## Problem 3.

Suppose the height of a flying bird above the ground at position $x$ is given by $y=h(x)=$ $\sqrt{4-x^{2}}$. (So that the $x$-axis lies along the ground, and the bird's height is given by $y$.)
(a) [12pts.] Find the average height of the bird as it flies from $(0,2)$ to $(2,0)$.
(b) [13pts.] Find the distance travelled by the bird along the same interval.

Problem 4. 15pts.
Find the volume of the solid generated by rotating the region bounded by $x=y^{2}+1$ and $x=2$ about the axis $y=-2$.

## Problem 5.

(a) [5pts.] Find the area of the region bounded by $y=\sin x, y=\cos x, x=0$, and $x=\frac{\pi}{4}$.
(b) [10pts.] Find the volume of the solid generated by rotating the region in (a) about the line $y=-1$.

Problem 6. 15pts.
Evaluate the following integral, if possible.

$$
\int_{1}^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} d x
$$

## Problem 7.

Extra credit: A group of engineers is building a parabolic reflector dish whose shape will be formed by rotating the curve $y=a x^{2}$ between $x=0 \mathrm{~m}$ and $x=1 \mathrm{~m}$ about the $y$-axis. What is the surface area of the resulting dish?

