S1012.001 Summer 2011
Calculus II

Final

Instructions: You have 90 minutes to complete the exam. There are seven problems, worth a total of 120 points. (Plus some extra credit in case you get bored.) Calculators and textbooks are not allowed. Provide the answers in the simplest possible form that does not require calculator use. (E.g. expressions like $\sqrt{13}$ are fine.) Show all of your work: if you only give the answer you will receive no credit, but conversely, partial credit will be given for partial solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	5	
2	20	
3	20	
4	25	
5	15	
6	15	
7	20	
8	0	
Total:	120	

Problem 1. *5pts.*

Evaluate the following integral.

$$\int \cos x \ln(\sin x) dx$$

Problem 2.

(a) [10pts.] Find the length of the curve $f(x) = \int_1^x \sqrt{\sqrt{t} - 1}dt$ for $1 \leq x \leq 16$.

(b) [10pts.] Find the surface area of the solid generated by rotating the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$ on $1 \leq x \leq 2$ about the y-axis.

Problem 3.

Let $x = e^t \cos(t), y = e^t \sin(t)$, for $0 \leq t \leq \pi$.

(a) [10pts.] Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) [10pts.] Find the exact length of the curve.

Problem 4.

(a) [5pts.] Sketch the curve $r = \sin(3\theta)$ for $0 \leq \theta \leq \pi$.

(b) [7pts.] Find the tangent line to the curve at $(1, \frac{\pi}{6})$.

(c) [8pts.] Find the area enclosed by a single loop of the curve.

(d) [5pts.] Find all points of intersection between the curve and $r = \frac{1}{2}$.

Problem 5.

(a) [5pts.] Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{1+2^n}{3^{n-1}}.$$

(b) [10pts.] For each of the series below, find some number n so that the nth partial sum of the series is within .001 of the actual sum of the series. It does not need to be the smallest such n. Justify your answer. (Do not attempt to compute this partial sum.)

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n!}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

Problem 6.

Determine whether the following series are conditionally convergent, absolutely convergent, or divergent. Clearly show that the hypotheses of any theorems you use are satisfied.

(a) [5pts.]

$$\sum_{n=1}^{\infty} \frac{\cos(n\pi/3)}{n!}$$

(b) [5pts.]

$$\sum_{n=1}^{\infty} \frac{n^2+4n+3}{5-3n^2}$$

(c) [5pts.]

$$\sum_{n=1}^{\infty} (-1)^n(\sqrt{n+1}-\sqrt{n})$$

Problem 7.

(a) [10pts.] Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{n^{\frac{1}{4}}}$$

(b) [10pts.] Recall that

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

Evaluate $\int \sin(x^2)dx$. (Note that this is impossible using the techniques of integration from the first half of the class. :)

Problem 8.

Extra Credit. Suppose you know that $\sum_{n=0}^{\infty} c_n(x-3)^n$ converges for $x=1$ but diverges for $x=7$. What, if anything, can you say about the following?

$$\sum_{n=0}^{\infty} c_n$$

$$\sum_{n=0}^{\infty} c_n 5^n$$

$$\sum_{n=0}^{\infty} c_n 2^n$$