Daily Homework Week 2

9. $\dfrac{5x+1}{(2x+1)(x-1)} = \dfrac{A}{2x+1} + \dfrac{B}{x-1}$. Multiply both sides by $(2x+1)(x-1)$ to get $5x+1 = A(x-1) + B(2x+1) \quad \Rightarrow$

$5x+1 = Ax - A + 2Bx + B \quad \Rightarrow \quad 5x+1 = (A+2B)x + (-A+B)$.

The coefficients of x must be equal and the constant terms are also equal, so $A + 2B = 5$ and

$-A + B = 1$. Adding these equations gives us $3B = 6 \quad \Leftrightarrow \quad B = 2$, and hence, $A = 1$. Thus,

$$\int \frac{5x+1}{(2x+1)(x-1)}\,dx = \int \left(\frac{1}{2x+1} + \frac{2}{x-1}\right) dx = \tfrac{1}{2}\ln|2x+1| + 2\ln|x-1| + C.$$

Another method: Substituting 1 for x in the equation $5x+1 = A(x-1) + B(2x+1)$ gives $6 = 3B \quad \Leftrightarrow \quad B = 2$.

Substituting $-\frac{1}{2}$ for x gives $-\frac{3}{2} = -\frac{3}{2}A \quad \Leftrightarrow \quad A = 1$.

17. Let $t = 0$ and $t = 12$ correspond to 9 AM and 9 PM, respectively.

$$T_{\text{ave}} = \tfrac{1}{12-0}\int_0^{12}\left[50 + 14\sin\tfrac{1}{12}\pi t\right]dt = \tfrac{1}{12}\left[50t - 14\cdot\tfrac{12}{\pi}\cos\tfrac{1}{12}\pi t\right]_0^{12}$$
$$= \tfrac{1}{12}\left[50\cdot 12 + 14\cdot\tfrac{12}{\pi} + 14\cdot\tfrac{12}{\pi}\right] = \left(50 + \tfrac{28}{\pi}\right)\,^\circ\text{F} \approx 59\,^\circ\text{F}$$

4. $A = \displaystyle\int_0^3 \left[(2y - y^2) - (y^2 - 4y)\right] dy = \int_0^3 (-2y^2 + 6y)\,dy = \left[-\tfrac{2}{3}y^3 + 3y^2\right]_0^3 = (-18 + 27) - 0 = 9$

2. A cross-section is a disk with radius $1 - x^2$, so its area is

$A(x) = \pi(1 - x^2)^2$.

$$V = \int_{-1}^{1} A(x)\,dx = \int_{-1}^{1} \pi(1-x^2)^2\,dx$$
$$= 2\pi\int_0^1 (1 - 2x^2 + x^4)\,dx = 2\pi\left[x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5\right]_0^1$$
$$= 2\pi\left(1 - \tfrac{2}{3} + \tfrac{1}{5}\right) = 2\pi\left(\tfrac{8}{15}\right) = \tfrac{16}{15}\pi$$