3. The area under the graph of $y = 1/x^3 = x^{-3}$ between $x = 1$ and $x = t$ is

$A(t) = \int_1^t x^{-3}\,dx = \left[-\frac{1}{2}x^{-2}\right]_1^t = -\frac{1}{2}t^{-2} - \left(-\frac{1}{2}\right) = \frac{1}{2} - 1/(2t^2)$. So the area for $1 \le x \le 10$ is

$A(10) = 0.5 - 0.005 = 0.495$, the area for $1 \le x \le 100$ is $A(100) = 0.5 - 0.00005 = 0.49995$, and the area for

$1 \le x \le 1000$ is $A(1000) = 0.5 - 0.0000005 = 0.4999995$. The total area under the curve for $x \ge 1$ is

$\lim\limits_{t\to\infty} A(t) = \lim\limits_{t\to\infty} \left[\frac{1}{2} - 1/(2t^2)\right] = \frac{1}{2}$.

7. $y = 1 + 6x^{3/2} \quad \Rightarrow \quad dy/dx = 9x^{1/2} \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + 81x$. So

$$L = \int_0^1 \sqrt{1 + 81x}\,dx = \int_1^{82} u^{1/2}\left(\tfrac{1}{81}\,du\right) \quad \begin{bmatrix} u = 1 + 81x, \\ du = 81\,dx \end{bmatrix} \quad = \frac{1}{81} \cdot \frac{2}{3}\left[u^{3/2}\right]_1^{82} = \frac{2}{243}\left(82\sqrt{82} - 1\right)$$