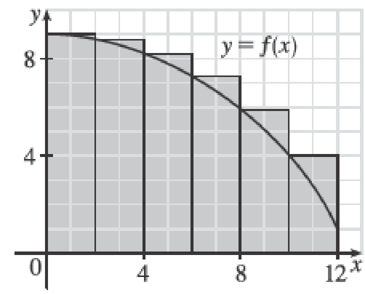
2. (a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x \qquad [\Delta x = \frac{12-0}{6} = 2]$

$= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$

$= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$

$\approx 2(9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1)$

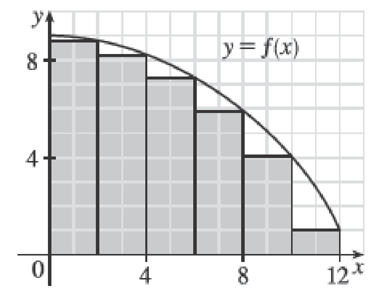
$= 2(43.3) = 86.6$



(ii) $R_6 = L_6 + 2 \cdot f(12) - 2 \cdot f(0)$

$\approx 86.6 + 2(1) - 2(9) = 70.6$

[Add area of rightmost lower rectangle
and subtract area of leftmost upper rectangle.]

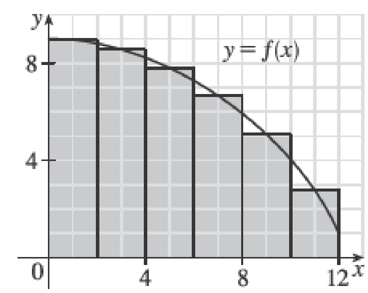


(iii) $M_6 = \sum_{i=1}^{6} f(x_i^*)\,\Delta x$

$= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$

$\approx 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$

$= 2(39.7) = 79.4$



(b) Since f is *decreasing*, we obtain an *overestimate* by using *left* endpoints; that is, L_6.

(c) Since f is *decreasing*, we obtain an *underestimate* by using *right* endpoints; that is, R_6.

(d) M_6 gives the best estimate, since the area of each rectangle appears to be closer to the true area than the overestimates and underestimates in L_6 and R_6.

14. (a) $d \approx L_5 = (30 \text{ ft/s})(12 \text{ s}) + 28 \cdot 12 + 25 \cdot 12 + 22 \cdot 12 + 24 \cdot 12$

$= (30 + 28 + 25 + 22 + 24) \cdot 12 = 129 \cdot 12 = 1548 \text{ ft}$

(b) $d \approx R_5 = (28 + 25 + 22 + 24 + 27) \cdot 12 = 126 \cdot 12 = 1512 \text{ ft}$

(c) The estimates are neither lower nor upper estimates since v is neither an increasing nor a decreasing function of t.

33. (a) Think of $\int_0^2 f(x)\,dx$ as the area of a trapezoid with bases 1 and 3 and height 2. The area of a trapezoid is $A = \frac{1}{2}(b+B)h$, so $\int_0^2 f(x)\,dx = \frac{1}{2}(1+3)2 = 4$.

(b) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx$

trapezoid rectangle triangle

$= \frac{1}{2}(1+3)2 + 3\cdot 1 + \frac{1}{2}\cdot 2\cdot 3 = 4+3+3 = 10$

(c) $\int_5^7 f(x)\,dx$ is the negative of the area of the triangle with base 2 and height 3. $\int_5^7 f(x)\,dx = -\frac{1}{2}\cdot 2\cdot 3 = -3$.

(d) $\int_7^9 f(x)\,dx$ is the negative of the area of a trapezoid with bases 3 and 2 and height 2, so it equals $-\frac{1}{2}(B+b)h = -\frac{1}{2}(3+2)2 = -5$. Thus,

$\int_0^9 f(x)\,dx = \int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx + \int_7^9 f(x)\,dx = 10 + (-3) + (-5) = 2.$

47. $\int_{-2}^2 f(x)\,dx + \int_2^5 f(x)\,dx - \int_{-2}^{-1} f(x)\,dx = \int_{-2}^5 f(x)\,dx + \int_{-1}^{-2} f(x)\,dx$ [by Property 5 and reversing limits]

$= \int_{-1}^5 f(x)\,dx$ [Property 5]

48. $\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = 12 - 3.6 = 8.4$

8. $f(t) = e^{t^2-t}$ and $g(x) = \int_3^x e^{t^2-t}\,dt$, so by FTC1, $g'(x) = f(x) = e^{x^2-x}$.

17. Let $w = 1-3x$. Then $\frac{dw}{dx} = -3$. Also, $\frac{dy}{dx} = \frac{dy}{dw}\frac{dw}{dx}$, so

$$y' = \frac{d}{dx}\int_{1-3x}^1 \frac{u^3}{1+u^2}\,du = \frac{d}{dw}\int_w^1 \frac{u^3}{1+u^2}\,du\cdot\frac{dw}{dx} = -\frac{d}{dw}\int_1^w \frac{u^3}{1+u^2}\,du\cdot\frac{dw}{dx} = -\frac{w^3}{1+w^2}(-3) = \frac{3(1-3x)^3}{1+(1-3x)^2}$$

30. $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2-y-1)\,dy = \left[\frac{2}{3}y^3 - \frac{1}{2}y^2 - y\right]_0^2 = \left(\frac{16}{3}-2-2\right) - 0 = \frac{4}{3}$

32. $\int_0^{\pi/4} \sec\theta\,\tan\theta\,d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2}-1$

40. $\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 (4u^{-3}+u^{-1})\,du = \left[\frac{4}{-2}u^{-2} + \ln|u|\right]_1^2 = \left[\frac{-2}{u^2} + \ln u\right]_1^2 = \left(-\frac{1}{2}+\ln 2\right) - (-2+\ln 1) = \frac{3}{2} + \ln 2$

27. $\int_0^\pi (5e^x + 3\sin x)\,dx = [5e^x - 3\cos x]_0^\pi = [5e^\pi - 3(-1)] - [5(1) - 3(1)] = 5e^\pi + 1$

35. $\int_0^1 (x^{10}+10^x)\,dx = \left[\frac{x^{11}}{11} + \frac{10^x}{\ln 10}\right]_0^1 = \left(\frac{1}{11} + \frac{10}{\ln 10}\right) - \left(0 + \frac{1}{\ln 10}\right) = \frac{1}{11} + \frac{9}{\ln 10}$

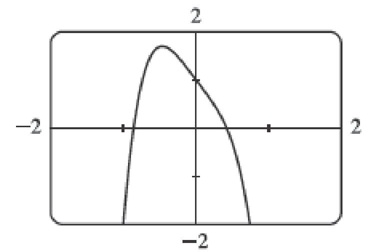
37. $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4}\left(\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\right)d\theta = \int_0^{\pi/4}(\sec^2\theta + 1)\,d\theta$

$= \left[\tan\theta + \theta\right]_0^{\pi/4} = \left(\tan\frac{\pi}{4} + \frac{\pi}{4}\right) - (0+0) = 1 + \frac{\pi}{4}$

39. $\int_1^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \int_1^{64}\left(\frac{1}{x^{1/2}}+\frac{x^{1/3}}{x^{1/2}}\right)dx = \int_1^{64}\left(x^{-1/2}+x^{(1/3)-(1/2)}\right)dx = \int_1^{64}(x^{-1/2}+x^{-1/6})\,dx$

$$= \left[2x^{1/2}+\tfrac{6}{5}x^{5/6}\right]_1^{64} = \left(16+\tfrac{192}{5}\right)-\left(2+\tfrac{6}{5}\right) = 14+\tfrac{186}{5} = \tfrac{256}{5}$$

47. The graph shows that $y = 1-2x-5x^4$ has x-intercepts at $x = a \approx -0.86$ and at $x = b \approx 0.42$. So the area of the region that lies under the curve and above the x-axis is



$$\int_a^b (1-2x-5x^4)\,dx = \left[x-x^2-x^5\right]_a^b$$
$$= (b-b^2-b^5)-(a-a^2-a^5)$$
$$\approx 1.36$$

13. Let $u = 5-3x$. Then $du = -3\,dx$ and $dx = -\frac{1}{3}\,du$, so

$$\int \frac{dx}{5-3x} = \int \frac{1}{u}\left(-\tfrac{1}{3}\,du\right) = -\tfrac{1}{3}\ln|u|+C = -\tfrac{1}{3}\ln|5-3x|+C.$$

25. Let $u = 1+e^x$. Then $du = e^x\,dx$, so $\int e^x\sqrt{1+e^x}\,dx = \int\sqrt{u}\,du = \frac{2}{3}u^{3/2}+C = \frac{2}{3}(1+e^x)^{3/2}+C$.

Or: Let $u = \sqrt{1+e^x}$. Then $u^2 = 1+e^x$ and $2u\,du = e^x\,dx$, so

$\int e^x\sqrt{1+e^x}\,dx = \int u\cdot 2u\,du = \frac{2}{3}u^3+C = \frac{2}{3}(1+e^x)^{3/2}+C$.

42. Let $u = \cos t$. Then $du = -\sin t\,dt$ and $\sin t\,dt = -du$, so

$\int \sin t\,\sec^2(\cos t)\,dt = \int \sec^2 u\cdot(-du) = -\tan u + C = -\tan(\cos t)+C$.

44. Let $u = x^2$. Then $du = 2x\,dx$, so $\displaystyle\int \frac{x}{1+x^4}\,dx = \int\frac{\frac{1}{2}\,du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}(x^2)+C$.

86. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 xf(x^2)\,dx = \int_0^9 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2$.

11. Let $u = \arctan 4t$, $dv = dt \quad\Rightarrow\quad du = \dfrac{4}{1+(4t)^2}\,dt = \dfrac{4}{1+16t^2}\,dt$, $v = t$. Then

$$\int \arctan 4t\,dt = t\arctan 4t - \int\frac{4t}{1+16t^2}\,dt = t\arctan 4t - \frac{1}{8}\int\frac{32t}{1+16t^2}\,dt = t\arctan 4t - \tfrac{1}{8}\ln(1+16t^2)+C.$$

17. First let $u = \sin 3\theta$, $dv = e^{2\theta}\,d\theta \quad\Rightarrow\quad du = 3\cos 3\theta\,d\theta$, $v = \frac{1}{2}e^{2\theta}$. Then

$I = \int e^{2\theta}\sin 3\theta\,d\theta = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{2}\int e^{2\theta}\cos 3\theta\,d\theta$. Next let $U = \cos 3\theta$, $dV = e^{2\theta}\,d\theta \quad\Rightarrow\quad dU = -3\sin 3\theta\,d\theta$, $V = \frac{1}{2}e^{2\theta}$ to get $\int e^{2\theta}\cos 3\theta\,d\theta = \frac{1}{2}e^{2\theta}\cos 3\theta + \frac{3}{2}\int e^{2\theta}\sin 3\theta\,d\theta$. Substituting in the previous formula gives

$I = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta - \frac{9}{4}\int e^{2\theta}\sin 3\theta\,d\theta = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta - \frac{9}{4}I \quad\Rightarrow$

$\frac{13}{4}I = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta + C_1$. Hence, $I = \frac{1}{13}e^{2\theta}(2\sin 3\theta - 3\cos 3\theta)+C$, where $C = \frac{4}{13}C_1$.

29. Let $u = y$, $dv = \dfrac{dy}{e^{2y}} = e^{-2y}dy \quad \Rightarrow \quad du = dy$, $v = -\frac{1}{2}e^{-2y}$. Then

$$\int_0^1 \frac{y}{e^{2y}}\,dy = \left[-\tfrac{1}{2}ye^{-2y}\right]_0^1 + \tfrac{1}{2}\int_0^1 e^{-2y}dy = \left(-\tfrac{1}{2}e^{-2} + 0\right) - \tfrac{1}{4}\left[e^{-2y}\right]_0^1 = -\tfrac{1}{2}e^{-2} - \tfrac{1}{4}e^{-2} + \tfrac{1}{4} = \tfrac{1}{4} - \tfrac{3}{4}e^{-2}.$$

33. Let $u = \ln(\sin x)$, $dv = \cos x\,dx \quad \Rightarrow \quad du = \dfrac{\cos x}{\sin x}\,dx$, $v = \sin x$. Then

$I = \int \cos x \ln(\sin x)\,dx = \sin x \ln(\sin x) - \int \cos x\,dx = \sin x \ln(\sin x) - \sin x + C.$

Another method: Substitute $t = \sin x$, so $dt = \cos x\,dx$. Then $I = \int \ln t\,dt = t\ln t - t + C$ (see Example 2) and so

$I = \sin x\,(\ln \sin x - 1) + C.$

48. (a) Let $u = \cos^{n-1} x$, $dv = \cos x\,dx \quad \Rightarrow \quad du = -(n-1)\cos^{n-2} x\,\sin x\,dx$, $v = \sin x$ in (2):

$$\begin{aligned}\int \cos^n x\,dx &= \cos^{n-1} x\,\sin x + (n-1)\int \cos^{n-2} x\,\sin^2 x\,dx \\ &= \cos^{n-1} x\,\sin x + (n-1)\int \cos^{n-2} x\,(1-\cos^2 x)dx \\ &= \cos^{n-1} x\,\sin x + (n-1)\int \cos^{n-2} x\,dx - (n-1)\int \cos^n x\,dx\end{aligned}$$

Rearranging terms gives $n\int \cos^n x\,dx = \cos^{n-1} x\,\sin x + (n-1)\int \cos^{n-2} x\,dx$ or

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x\,\sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

(b) Take $n = 2$ in part (a) to get $\int \cos^2 x\,dx = \frac{1}{2}\cos x\,\sin x + \frac{1}{2}\int 1\,dx = \dfrac{x}{2} + \dfrac{\sin 2x}{4} + C.$

(c) $\int \cos^4 x\,dx = \frac{1}{4}\cos^3 x\,\sin x + \frac{3}{4}\int \cos^2 x\,dx = \frac{1}{4}\cos^3 x\,\sin x + \frac{3}{8}x + \frac{3}{16}\sin 2x + C$

11. $\int_0^{\pi/2} \sin^2 x\,\cos^2 x\,dx = \int_0^{\pi/2} \frac{1}{4}(4\sin^2 x\,\cos^2 x)\,dx = \int_0^{\pi/2} \frac{1}{4}(2\sin x\,\cos x)^2 dx = \frac{1}{4}\int_0^{\pi/2} \sin^2 2x\,dx$

$$= \tfrac{1}{4}\int_0^{\pi/2} \tfrac{1}{2}(1-\cos 4x)\,dx = \tfrac{1}{8}\int_0^{\pi/2}(1-\cos 4x)\,dx = \tfrac{1}{8}\left[x - \tfrac{1}{4}\sin 4x\right]_0^{\pi/2} = \tfrac{1}{8}\left(\tfrac{\pi}{2}\right) = \tfrac{\pi}{16}$$

15. $$\int \frac{\cos^5\alpha}{\sqrt{\sin\alpha}}\,d\alpha = \int \frac{\cos^4\alpha}{\sqrt{\sin\alpha}}\cos\alpha\,d\alpha = \int \frac{\left(1-\sin^2\alpha\right)^2}{\sqrt{\sin\alpha}}\cos\alpha\,d\alpha \overset{s}{=} \int \frac{(1-u^2)^2}{\sqrt{u}}\,du$$

$$= \int \frac{1-2u^2+u^4}{u^{1/2}}\,du = \int (u^{-1/2} - 2u^{3/2} + u^{7/2})\,du = 2u^{1/2} - \tfrac{4}{5}u^{5/2} + \tfrac{2}{9}u^{9/2} + C$$

$$= \tfrac{2}{45}u^{1/2}(45 - 18u^2 + 5u^4) + C = \tfrac{2}{45}\sqrt{\sin\alpha}\,(45 - 18\sin^2\alpha + 5\sin^4\alpha) + C$$

19. $$\int \frac{\cos x + \sin 2x}{\sin x}\,dx = \int \frac{\cos x + 2\sin x\,\cos x}{\sin x}\,dx = \int \frac{\cos x}{\sin x}\,dx + \int 2\cos x\,dx \overset{s}{=} \int \frac{1}{u}\,du + 2\sin x$$

$$= \ln|u| + 2\sin x + C = \ln|\sin x| + 2\sin x + C$$

Or: Use the formula $\int \cot x\,dx = \ln|\sin x| + C$.

38. $\int \csc^4 x\,\cot^6 x\,dx = \int \cot^6 x\,(\cot^2 x + 1)\,\csc^2 x\,dx = \int u^6(u^2+1)\cdot(-du) \qquad [u = \cot x,\ du = -\csc^2 x\,dx]$

$$= \int u^6(u^2+1)\cdot(-du) \qquad [u = \cot x,\ du = -\csc^2 x\,dx]$$

$$= \int (-u^8 - u^6)\,du = -\tfrac{1}{9}u^9 - \tfrac{1}{7}u^7 + C = -\tfrac{1}{9}\cot^9 x - \tfrac{1}{7}\cot^7 x + C$$

41. $\int \sin 8x \, \cos 5x \, dx \overset{2a}{=} \int \frac{1}{2}[\sin(8x-5x)+\sin(8x+5x)]\,dx = \frac{1}{2}\int \sin 3x\,dx + \frac{1}{2}\int \sin 13x\,dx$

$$= -\tfrac{1}{6}\cos 3x - \tfrac{1}{26}\cos 13x + C$$

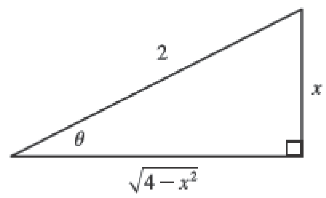
68. $\int_{-\pi}^{\pi} \sin mx \, \sin nx \, dx = \int_{-\pi}^{\pi} \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]\,dx.$

If $m \neq n$, this is equal to $\dfrac{1}{2}\left[\dfrac{\sin(m-n)x}{m-n} - \dfrac{\sin(m+n)x}{m+n}\right]_{-\pi}^{\pi} = 0.$

If $m = n$, we get $\int_{-\pi}^{\pi} \frac{1}{2}[1-\cos(m+n)x]\,dx = \left[\frac{1}{2}x\right]_{-\pi}^{\pi} - \left[\dfrac{\sin(m+n)x}{2(m+n)}\right]_{-\pi}^{\pi} = \pi - 0 = \pi.$

1. Let $x = 2\sin\theta$, where $-\pi/2 \le \theta \le \pi/2$. Then $dx = 2\cos\theta\,d\theta$ and

$\sqrt{4-x^2} = \sqrt{4-4\sin^2\theta} = \sqrt{4\cos^2\theta} = 2\,|\cos\theta| = 2\cos\theta.$



Thus, $\displaystyle\int \frac{dx}{x^2\sqrt{4-x^2}} = \int \frac{2\cos\theta}{4\sin^2\theta(2\cos\theta)}\,d\theta = \frac{1}{4}\int \csc^2\theta\,d\theta$

$$= -\frac{1}{4}\cot\theta + C = -\frac{\sqrt{4-x^2}}{4x} + C \quad \text{[see figure]}$$

16. Let $x = \frac{1}{3}\sec\theta$, so $dx = \frac{1}{3}\sec\theta\,\tan\theta\,d\theta$, $x = \sqrt{2}/3 \;\Rightarrow\; \theta = \frac{\pi}{4}$, $x = \frac{2}{3} \;\Rightarrow\; \theta = \frac{\pi}{3}$. Then

$$\int_{\sqrt{2}/3}^{2/3} \frac{dx}{x^5\sqrt{9x^2-1}} = \int_{\pi/4}^{\pi/3} \frac{\frac{1}{3}\sec\theta\,\tan\theta\,d\theta}{\left(\frac{1}{3}\right)^5\sec^5\theta\,\tan\theta} = 3^4\int_{\pi/4}^{\pi/3}\cos^4\theta\,d\theta = 81\int_{\pi/4}^{\pi/3}\left[\tfrac{1}{2}(1+\cos 2\theta)\right]^2\,d\theta$$

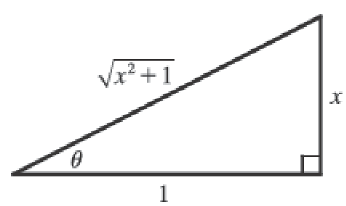
$$= \tfrac{81}{4}\int_{\pi/4}^{\pi/3}(1+2\cos 2\theta+\cos^2 2\theta)\,d\theta = \tfrac{81}{4}\int_{\pi/4}^{\pi/3}\left[1+2\cos 2\theta+\tfrac{1}{2}(1+\cos 4\theta)\right]d\theta$$

$$= \tfrac{81}{4}\int_{\pi/4}^{\pi/3}\left(\tfrac{3}{2}+2\cos 2\theta+\tfrac{1}{2}\cos 4\theta\right)d\theta = \tfrac{81}{4}\left[\tfrac{3}{2}\theta+\sin 2\theta+\tfrac{1}{8}\sin 4\theta\right]_{\pi/4}^{\pi/3}$$

$$= \tfrac{81}{4}\left[\left(\tfrac{\pi}{2}+\tfrac{\sqrt{3}}{2}-\tfrac{\sqrt{3}}{16}\right)-\left(\tfrac{3\pi}{8}+1+0\right)\right] = \tfrac{81}{4}\left(\tfrac{\pi}{8}+\tfrac{7}{16}\sqrt{3}-1\right)$$

22. Let $x = \tan\theta$, where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then $dx = \sec^2\theta\,d\theta$,

$\sqrt{x^2+1} = \sec\theta$ and $x = 0 \;\Rightarrow\; \theta = 0$, $x = 1 \;\Rightarrow\; \theta = \frac{\pi}{4}$, so

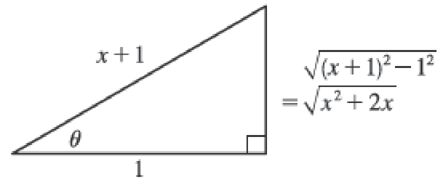


$\int_0^1 \sqrt{x^2+1}\,dx = \int_0^{\pi/4}\sec\theta\,\sec^2\theta\,d\theta = \int_0^{\pi/4}\sec^3\theta\,d\theta$

$$= \tfrac{1}{2}\Big[\sec\theta\,\tan\theta + \ln|\sec\theta+\tan\theta|\Big]_0^{\pi/4} \quad \text{[by Example 7.2.8]}$$

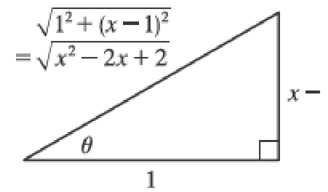
$$= \tfrac{1}{2}\left[\sqrt{2}\cdot 1 + \ln\left(1+\sqrt{2}\right) - 0 - \ln(1+0)\right] = \tfrac{1}{2}\left[\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right]$$

27. $x^2 + 2x = (x^2 + 2x + 1) - 1 = (x+1)^2 - 1$. Let $x + 1 = 1\sec\theta$, so $dx = \sec\theta\,\tan\theta\,d\theta$ and $\sqrt{x^2+2x} = \tan\theta$. Then



$$\int \sqrt{x^2+2x}\,dx = \int \tan\theta\,(\sec\theta\,\tan\theta\,d\theta) = \int \tan^2\theta\,\sec\theta\,d\theta$$
$$= \int (\sec^2\theta - 1)\,\sec\theta\,d\theta = \int \sec^3\theta\,d\theta - \int \sec\theta\,d\theta$$
$$= \tfrac{1}{2}\sec\theta\,\tan\theta + \tfrac{1}{2}\ln|\sec\theta + \tan\theta| - \ln|\sec\theta+\tan\theta| + C$$
$$= \tfrac{1}{2}\sec\theta\,\tan\theta - \tfrac{1}{2}\ln|\sec\theta + \tan\theta| + C = \tfrac{1}{2}(x+1)\sqrt{x^2+2x} - \tfrac{1}{2}\ln\left|x+1+\sqrt{x^2+2x}\right| + C$$

28. $x^2 - 2x + 2 = (x^2 - 2x + 1) + 1 = (x-1)^2 + 1$. Let $x - 1 = 1\tan\theta$, so $dx = \sec^2\theta\,d\theta$ and $\sqrt{x^2-2x+2} = \sec\theta$. Then



$$\int \frac{x^2+1}{(x^2-2x+2)^2}\,dx = \int \frac{(\tan\theta+1)^2+1}{\sec^4\theta}\,\sec^2\theta\,d\theta$$
$$= \int \frac{\tan^2\theta + 2\tan\theta + 2}{\sec^2\theta}\,d\theta$$
$$= \int (\sin^2\theta + 2\sin\theta\,\cos\theta + 2\cos^2\theta)\,d\theta = \int (1 + 2\sin\theta\,\cos\theta + \cos^2\theta)\,d\theta$$
$$= \int \left[1 + 2\sin\theta\,\cos\theta + \tfrac{1}{2}(1+\cos 2\theta)\right]\,d\theta = \int \left(\tfrac{3}{2} + 2\sin\theta\,\cos\theta + \tfrac{1}{2}\cos 2\theta\right)\,d\theta$$
$$= \tfrac{3}{2}\theta + \sin^2\theta + \tfrac{1}{4}\sin 2\theta + C = \tfrac{3}{2}\theta + \sin^2\theta + \tfrac{1}{2}\sin\theta\,\cos\theta + C$$
$$= \frac{3}{2}\tan^{-1}\left(\frac{x-1}{1}\right) + \frac{(x-1)^2}{x^2-2x+2} + \frac{1}{2}\frac{x-1}{\sqrt{x^2-2x+2}}\frac{1}{\sqrt{x^2-2x+2}} + C$$
$$= \frac{3}{2}\tan^{-1}(x-1) + \frac{2(x^2-2x+1)+x-1}{2(x^2-2x+2)} + C = \frac{3}{2}\tan^{-1}(x-1) + \frac{2x^2-3x+1}{2(x^2-2x+2)} + C$$

We can write the answer as

$$\frac{3}{2}\tan^{-1}(x-1) + \frac{(2x^2-4x+4)+x-3}{2(x^2-2x+2)} + C = \frac{3}{2}\tan^{-1}(x-1) + 1 + \frac{x-3}{2(x^2-2x+2)} + C$$
$$= \frac{3}{2}\tan^{-1}(x-1) + \frac{x-3}{2(x^2-2x+2)} + C_1, \text{ where } C_1 = 1 + C$$