32. $\int_0^{\pi/4} \sec\theta \, \tan\theta \, d\theta = [\sec\theta]_0^{\pi/4} = \sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$

7. Let $u = x^2$. Then $du = 2x\,dx$ and $x\,dx = \frac{1}{2}\,du$, so $\int x\sin(x^2)\,dx = \int \sin u\left(\frac{1}{2}\,du\right) = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$.

8. First let $u = t^2$, $dv = \sin\beta t\,dt \quad \Rightarrow \quad du = 2t\,dt$, $v = -\frac{1}{\beta}\cos\beta t$. Then by Equation 2,

$I = \int t^2 \sin\beta t\,dt = -\frac{1}{\beta}t^2\cos\beta t - \int -\frac{2}{\beta}t\cos\beta t\,dt$. Next let $U = t$, $dV = \cos\beta t\,dt \quad \Rightarrow \quad dU = dt$,

$V = \dfrac{1}{\beta}\sin\beta t$, so $\int t\cos\beta t\,dt = \dfrac{1}{\beta}t\sin\beta t - \displaystyle\int \frac{1}{\beta}\sin\beta t\,dt = \frac{1}{\beta}t\sin\beta t + \frac{1}{\beta^2}\cos\beta t$. Thus,

$$I = -\frac{1}{\beta}t^2\cos\beta t + \frac{2}{\beta}\left(\frac{1}{\beta}t\sin\beta t + \frac{1}{\beta^2}\cos\beta t\right) + C = -\frac{1}{\beta}t^2\cos\beta t + \frac{2}{\beta^2}t\sin\beta t + \frac{2}{\beta^3}\cos\beta t + C.$$

6. Let $y = \sqrt{x}$, so that $dy = \dfrac{1}{2\sqrt{x}}\,dx$ and $dx = 2y\,dy$. Then

$$\begin{aligned}\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}}\,dx &= \int \frac{\sin^3 y}{y}(2y\,dy) = 2\int \sin^3 y\,dy = 2\int \sin^2 y\,\sin y\,dy = 2\int (1-\cos^2 y)\,\sin y\,dy \\ &\overset{c}{=} 2\int (1-u^2)(-du) = 2\int (u^2-1)\,du = 2\left(\tfrac{1}{3}u^3 - u\right) + C = \tfrac{2}{3}\cos^3 y - 2\cos y + C \\ &= \tfrac{2}{3}\cos^3(\sqrt{x}) - 2\cos\sqrt{x} + C\end{aligned}$$