Solutions: Practice Problems

1) Use the Handshaking Lemma:

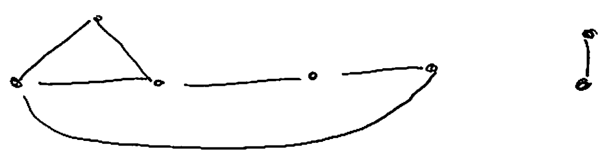
$$2(\text{size}) = \sum_{u \in V(G)} \deg u$$

$$68 = 5 \cdot 4 + 6 \cdot 5 + (20-5-6) \cdot x$$

$$\Rightarrow x = 2$$

2) a) No. Irregular graphs do not exist.

b) yes (by Havel-Hakimi):



c) No, by Havel-Hakimi

d) No, graphs must have even # of odd deg. vertices.

3) No. To see this note that the avg deg of G is

$$\frac{1}{n} \sum_{u \in V(G)} \deg u = \frac{2m}{n}$$

where $m = |E(G)|$. If we delete a vertex of deg Δ then the avg deg is now

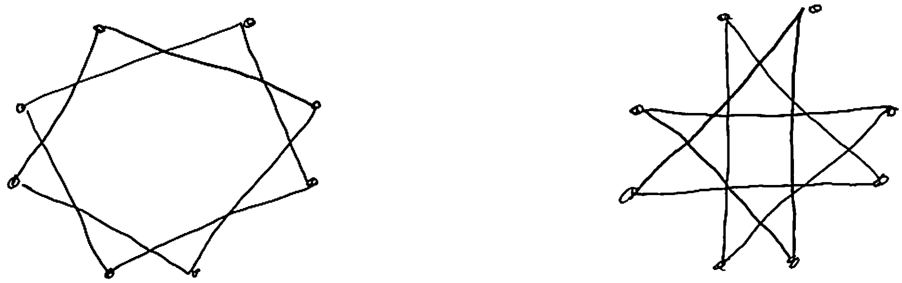
$$\frac{1}{n-1} \left(\sum_{u \in V(G)} \deg u - \Delta \right) = \frac{2m - \Delta}{n-1}.$$

Now $\frac{2m-\Delta}{n-1} \le \frac{2m}{n}$ iff $(2m-\Delta)n \le 2m(n-1)$

iff $2m \le \Delta n$ iff $\frac{2m}{n} \le \Delta$.

But this last statement must be true!

4) Take graph complements:



These are clearly not $\cong$ so the orig. graphs are <u>not</u> i'sm.

5) False. Consider P_n for $n \geq 4$.

6) This graph is regular. If w is an arbitrary word in G_n then it has exactly n letters. Every adj word must differ in exactly 2 (of the n) letters. Thus w has $\binom{n}{2}$ n'bors.

In other words, G_n is $\binom{n}{2}$-regular.

7) Let G be a graph w/ min deg $\delta(G)$. Now consider a <u>longest</u> path

$$P = (u_0 \ldots u_k)$$

in G. We may assume, for a contr. that $k < \delta(G)$. Now consider that $\deg u_k \geq \delta(G)$. So u_k must have a n'bor x where

$$x \notin \{\underbrace{u_0 \ldots u_{k-1}}_{k \text{ vertices.}}\}$$

Then $P' = (u_0 \ldots u_k\, x)$ is a longer path $\Rightarrow\Leftarrow$.

$\therefore$ $k \geq \delta(G)$.

8) Let $G_1, \cdots, G_k$ be all the cntd comp of G.

If $G_1, \cdots, G_k$ are not trees, and n_i & m_i are the order & size of G_i, then

$$n_i \leq m_i.$$

This means that, $|V(G)| = n_1 + \cdots + n_k$

$$\leq m_1 + \cdots + m_k = |E(G)|$$

$\Rightarrow\Leftarrow$

This contradicts the fact that

$$|E(G)| \lneq |V(G)|.$$

$\Rightarrow$ some comp of G must be a tree.