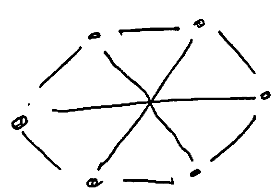
1) This situation may be modeled as a graph where there is an edge between two people iff they shake hands. If everyone shakes an odd # of hands, then every vertex has odd degree. By the Handshaking Lemma, this means the # of vertices (people) must be even.

2) a- Not graphical by Havel-Hakimi

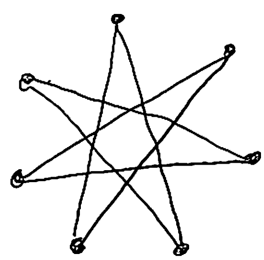
b- consider the Harary Graph

$H_{6,3}$ =



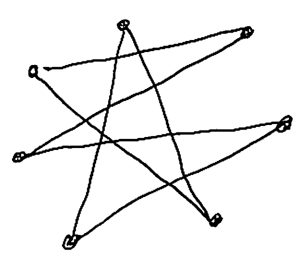
3) consider graph complements:

$\bar{A}$:



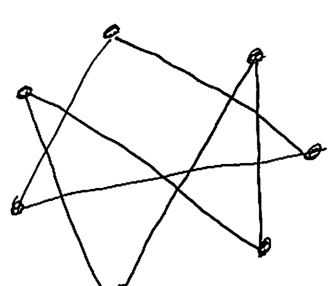
$\cong C_7$

$\bar{B}$:



$\cong C_7$

$\bar{C}$:



$\cong C_3 + C_4$

4) a- False, walks may repeat vertices

b- True, If T has order $n \geq 3$, then it has at least two leaves. As these have deg 1, then if T were regular it must be 1-reg. But the only cnt'd 1-reg graph is K_2 which does not have enough vertices.

$\therefore$ T is not regular.

c- We know that for any graph G w/ order $n \geq 2$

G is bipartite iff G has no odd cycle.

As trees have no cycles, then any tree w/ order $n \geq 2$ must be bipartite.

d- see HW 3 problem #1b.

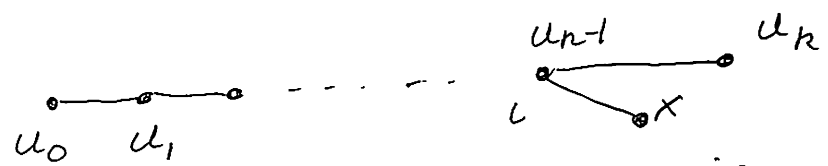
5) Let G be a cntd graph of order n. If T is the spanning tree obtained by pruning G then $|V(T)| = n$ and $|E(T)| = n-1$. So

$$\# \text{ edges deleted} = |E(G)| - (n-1)$$

which is indep of the pruning process.

6) see HW #3 problem #3.

7) Let P be a path of max length in G.



We know (from class) that u_k is a leaf. So $\deg u_{k-1} \geq 3$. Let x be some vertex not on P $\ni$ $x \sim u_{k-1}$. If x is not a leaf then it must have a n'bor y not on P (else we have a cycle). But then

$$(u_0, u_1, \ldots u_{k-1}\, x\, y)$$

is a longer path $\Rightarrow\Leftarrow$.

$\therefore$ x, u_k are leaves adj to u_{k-1}.