Math 428
Practice Problems #2

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1. Find an infinite set of graphs that have more cut-vertices than bridges.

2. Assume v is a cut-vertex of a graph G. Is v also a cut-vertex in $\overline{G}$?

3. Let G be a connected graph. Prove that G is nonseparable if and only if every pair of *adjacent* edges lie on a common cycle. (Note: Two edges are adjacent if they share a common endpoint.)

4. Let $e = xy$ be an edge in G and assume $\kappa(G) \geq 2$. Prove that $\kappa(G - e) \geq 2$ if and only if x, y lie on a common cycle in $G - e$.

5. Let G be a connected graph of order n. Prove that

$$n \geq \kappa(G)(\text{diam}(G) - 1).$$

   Recall that $\text{diam}(G) = \max_{x,y \in V(G)} d(x, y)$.

6. Fix G and H disjoint graphs. Let F be the graph obtained by adding a new vertex x that is adjacent to all the vertices in $G + H$. Prove that $\lambda(F) = \min(\delta(G), \delta(H)) + 1$.