$\begin{array}{c} {\rm Math~428} \\ {\rm Practice~Problems~\#1} \end{array}$

- 1. A graph G has order 20 and size 34 has 5 vertices with degree 4, 6 vertices of degree 5. The remaining vertices all have the same degree. What is this degree?
- 2. Determine whether the following sequences are graphical. If so, provide a graph with the appropriate degree sequence.
 - (a) 4,3,2,1,0
 - (b) 3,3,2,2,2,1,1
 - (c) 7, 6, 6, 5, 4, 3, 2, 1
 - (d) 3,3,3,3,3
- 3. Let G be a graph of order $n \ge 2$. If we delete from G a vertex v of degree $\Delta(G)$, can the average degree increase?
- 4. Determine whether the graphs below are isomorphic.



- 5. Prove or disprove. Every connected graph G of order 4 or more contains three distinct vertices u, v, and w such that G u, G v, and G w are connected.
- 6. Let G_n be the graph whose vertex set consists of all binary words of length n where two words are adjacent if and only if they differ in exactly two positions. Is G_n regular? If it is, find this common degree.
- 7. Prove that every graph G has a path of length $\delta(G)$.
- 8. Show that every graph with fewer edges than vertices has a connected component that is a tree.