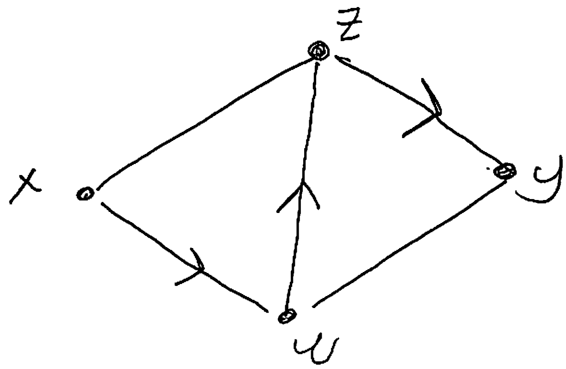
Solutions: HW 8

1) No. Be careful - this is NOT the same as thm 7.2. Consider:



If $P = (x, w, z, y)$ then $\nexists$ x-y path int. disj from P.

2) By symmetry it will suffice to find n int disj x-y paths where $x = \underbrace{0 \cdots 0}_{n}$ and y is any other pt in Q_n. To do this we proceed by induction on n.

Base case: $n = 1$

$Q_1 =$ 

Now assume the result for $n \geq 1$ and consider Q_{n+1}.
Let $x = \underbrace{0 \ldots 0}_{n+1}$ and $y \in V(Q_{n+1})$.

If y contains a 0 then wlog assume it is in the last position. Let x' & y' be the result of deleting the last positions. As $x', y' \in V(Q_n)$, then by induction $\exists$ n int disj x'-y' paths

$$P_i = (x' = x_1^{(i)}, x_2^{(i)}, \ldots, x_{k_i}^{(i)} = y')$$

Then

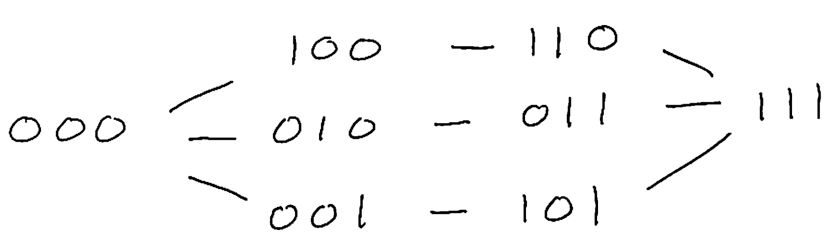
$$(\underbrace{x' \cdot 0}_{=x}, x_2^{(i)} \cdot 0, \ldots, x_{k_i}^{(i)} \cdot 0 = y) \quad (1 \leq i \leq n)$$

and

$$(x' \cdot 0, x' \cdot 1, x_2^{(1)} \cdot 1, \ldots, x_{k_1}^{(1)} \cdot 1, x_{k_1}^{(1)} \cdot 0 = y)$$

are $n+1$ int disj x-y paths in Q_n.

For the case $y = \underbrace{1 \ldots 1}_{n+1}$, i.e., it has no zeros then we illustrate w/ an ex. when $n = 3$:

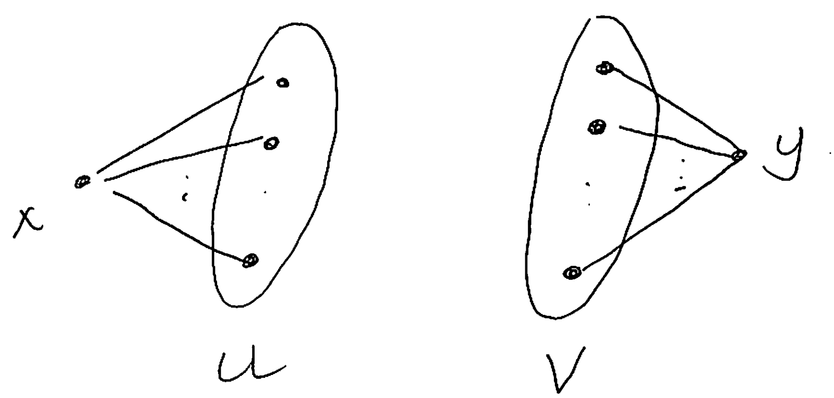


3) Consider the graph C_ℓ - the cycle on ℓ vertices. Then $\ell \geq 3$, $\kappa(C_\ell) = 2 < \ell$ and <u>every</u> set of ℓ vertices in C_ℓ (there is only one!) lie on a cycle.

4) By Whitney's thm $\exists$ at least 5 int disj u-v paths: $P_1 \cdots P_5$. As $w \neq v, u$ then at least 4 of them are guaranteed to <u>not</u> contain w. These 4 int-disj u-v paths constitute C and C' as desired.

5) Create a new graph G' from G by adding two new vertices x, y as shown:

G'



As $\kappa(G') \geq \ell$, it follows by Whitney's thm that G' contains exactly ℓ int disj x-y paths. By deleting x, y the desired result now follows.