Math 428
Graph Theory
Homework Set #7

Vertex & Edge Connectivity

1. In the Peterson graph PG find a smallest edge-cut and a smallest vertex-cut. Determine $\kappa(PG)$ and $\lambda(PG)$.

2. Let G be any graph of order n and size m. Assuming that $\kappa(G) \geq k$, for some integer k, show that $m \geq kn/2$.

3. Let G be any nontrivial graph with vertex v. Prove or disprove the following statement. Either $\kappa(G - v) = \kappa(G)$ or $\kappa(G - v) = \kappa(G) - 1$.

4. Give an example of a graph G with
   (a) $\kappa(G) = 2$, $\lambda(G) = 2$, and $\delta(G) = 3$.
   (b) $\kappa(G) = 1$, $\lambda(G) = 2$, and $\delta(G) = 3$.
   (c) $\kappa(G) \neq \lambda(G)$ and G is 4-regular.

5. Let G be a graph of order n and let k be an integer such that $1 \leq k \leq n - 1$. Prove that if $\delta(G) \geq (n + k - 2)/2$, then $\kappa(G) \geq k$.

6. Let G be a connected graph such that for every edge e there exists cycles C and C' whose only common edge is e. Prove that $\lambda(G) \geq 3$. Use this to show that $\lambda(PG) = 3$.

7. Let G be a graph with diameter 2. Show that if Σ is a smallest edge-cut, then at least one of the components of $G - \Sigma$ is K_1 or $K_\delta(G)$. (Hint: Consider the proof of Theorem 9.1.)

Optional Problems

1. Let $0 < a \leq b \leq c$ be positive integers. Prove that there exists a graph G with $\kappa(G) = a$, $\lambda(G) = b$, and $\delta(G) = c$.