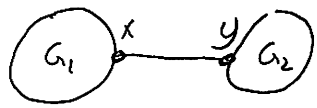
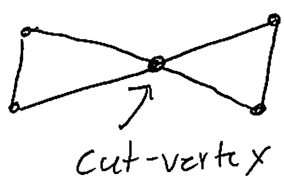
HW #6: Solutions

1) a- Assume $e = xy$ is a bridge in G. Then $G - e$ has two comp G_1 + G_2 as follows:



In G_1, x now has odd degree (as e has been deleted). Moreover all the other vertices in G_1 are even. This is impossible b/c no graph can have an odd # of odd degrees.

b -



2) a) same ex as #1b

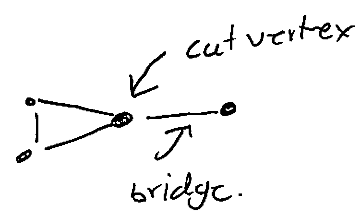
b) 

c) 

d) 

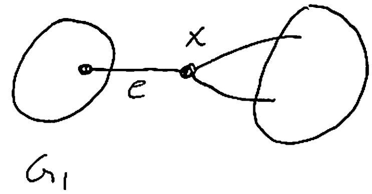
3) a) same ex as #1b

b) 

c) 

4) ($\Rightarrow$) As G is 3-regular, its order is ≥ 2.
Therefore thm 7.1 tells us that if $e = xy$ is a bridge then either x or y is a cut-vertex.

($\Leftarrow$) Assume G has a cut-vertex, x. Then $G - x$ has at least two components G_1 & G_2.
As x is adj to at least one vertex in each comp and $\deg x = 3$ we see that x must be adj to exactly one vertex in G_1 (wlog).



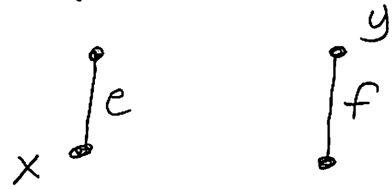
It is now clear that e is a bridge.

5) observe that if T is any tree and we delete a leaf, u, the result is still cntd. If T happens to be a <u>spanning tree</u> for some graph G, then since any $T - u \subseteq G - u$, and $T - u$ is cntd it follows that $G - u$ is cntd as well.
$\therefore$ u is not a cut-vertex.

If G is notrivial, then any spanning tree for G will also be nontrivial and hence have at least two leafs. By what we just proved these leaves cannot be cut-vertices.
$\therefore$ G has at least two non-cut-vertices.

6) To prove the reverse direction, we may assume our graph has at least two edges so $n \geq 3$. (Note the only cntd graph w/ 1 edge is K_2 which is nonseparable.)

Let $x, y \in V(G)$, and choose two edges e, f w/ $e \neq f$ so that e is incident to x and f is incident to y: (this is possible as $n \geq 3$).



By assumption, e & f lie on a common cycle. It now follows that x, y lie on a common cycle. By thm 7.2 we can conclude that G is nonseparable.