Math 428
Graph Theory
Homework Set #6

Cut-Vertices & Bridges

1. Let G be a graph such that every degree is even. Can G contain a bridge? A cut-vertex?

2. Give an example of a connected graph G with vertex v for each of the following:

   (a) v is on a cycle and v is a cut-vertex.

   (b) v is on a cycle and v is *not* a cut-vertex.

   (c) v is *not* on a cycle and v is a cut-vertex.

   (d) v is *not* on a cycle and v is *not* a cut-vertex.

   Conclude that there is no relationship between the idea of cut-vertices and cycles.

3. For each of the following, find a connected graph G, such that G has

   (a) a cut-vertex but no bridge.

   (b) a bridge but no cut-vertices.

   (c) exactly one bridge and exactly one cut-vertex.

4. Assume G is a 3-regular graph. Prove that G has a bridge if and only if it has a cut-vertex.

5. Let G be a nontrivial connected graph with spanning tree T. Prove that if u is a leaf of T, then u is not a cut-vertex of G. Conclude that every nontrivial connected graph has at least two vertices which are not cut-vertices.

6. In class we stated the following theorem but only proved the forward direction:

   Theorem (7.3)**.** *A connected graph G is nonseparable if and only if every pair of edges e, f lie on a common cycle.*

   Prove the reverse direction.