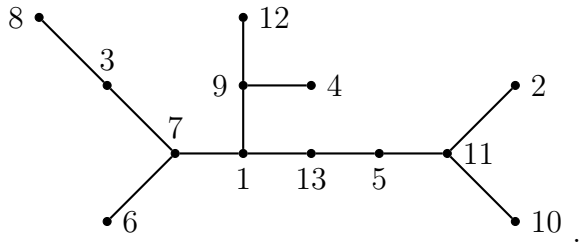
Math 428
Graph Theory
Homework Set #5

Prüfer Codes & Spanning Trees

1. (a) Find the labeled tree whose Prüfer code is $(1, 8, 1, 5, 2, 5)$.
   (b) Find the Prüfer code for the following tree



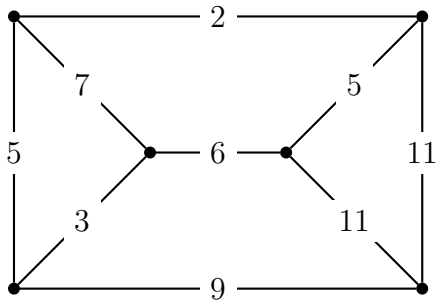
2. For each of the following explain your answer. (You do not need a rigorous proof.)
   (a) Which trees have constant Prüfer codes, i.e., codes of the form $a, a \ldots, a$?
   (b) Which trees have Prüfer codes that contain exactly two values.
   (c) Which trees have Prüfer codes with all distinct terms.

3. Give a formula, in terms of n, for the number of labeled trees of order n that have
   (a) exactly two leaves,
   (b) exactly $n - 2$ leaves.

4. Let w be the Prüfer code for a labeled tree T. Assume that some vertex u has label k. Show that k appears $(\deg u) - 1$ times in w.

5. (a) Which graphs have exactly one spanning tree?
   (b) For each $k \geq 3$, find some graph that has exactly k spanning trees.
   (c) Show no graph has exactly 2 spanning trees.

6. Use both Kruskal's and Prim's algorithm to find a minimum spanning tree for the following weighted graph



7. Prove that if a connected weighted graph has distinct edge weights, then it has exactly one minimum spanning tree. (Hint: Assume not and construct one using Kruskal's algorithm. Then consider the proof of Kurskal's algorithm.)

Optional Problems

1. Use Cayley's Formula to prove that the graph obtained from K_n by deleting one edge has $(n-2)n^{n-3}$ spanning trees.