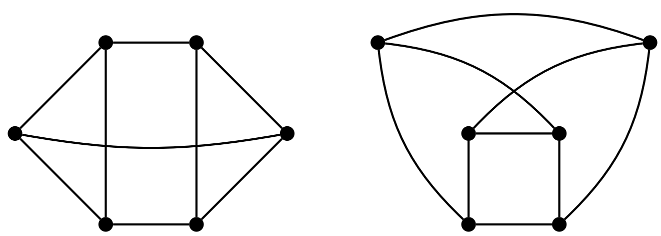
Math 428
Graph Theory
Homework Set #3

Graph Isomorphism

1. Find two non-isomorphic graphs of order 5 that have the same degree sequence. Explain why your graphs are not isomorphic.

2. Determine whether the graphs below are isomorphic (explain your reasoning):



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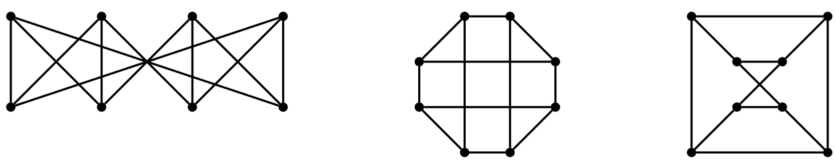
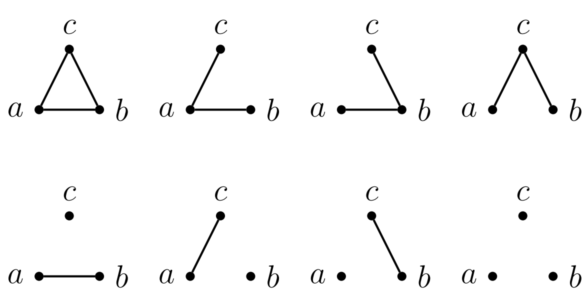


4. In class we mentioned that no formula for number of non-isomorphic graphs is known. There is a nice formula if we allow labeled graphs. Show that the number of distinct graphs on n labeled vertices is $2^{\frac{n(n-1)}{2}}$. For example the 8 distinct graphs on 3 vertices are:



5. Assume $\varphi : G \to H$ is an graph isomorphism and let $u \in V(G)$.

   a) Show that $\deg_G u = \deg_H \varphi(u)$.

   b) Let $G - u$ and $H - \varphi(u)$ be the graphs obtained by deleting the vertices u and $\varphi(u)$ (and all their edges). Show that $G - u \cong H - \varphi(u)$.

c) Conclude that if we delete all the vertices of degree d from both G and H, then the resulting graphs are still isomorphic.

Connected Graphs

1. (True of False)

   a) Must every closed walk contain a cycle? If so, prove it, otherwise give a counterexample.

   b) Let u and v be the only vertices in G with odd degree. Must G contain a $u-v$ path?

2. Let u be a vertex in a connected graph G. If $G-u$ is disconnected, show that u must be adjacent to some vertex in every component of $G-u$.

3. Show that if G is disconnected, then $\overline{G}$ is connected. Is the converse true?

4. Let G be the graph whose vertex set is the set of binary words of length ℓ where two words are adjacent iff they differ in *exactly* two positions. G consists of how many components?

5. Prove that the Peterson graph cannot have a cycle of length 4.

Optional Problems

1. Let G be self-complementary graph of order n, where $n \equiv 1 (\text{mod } 4)$. Prove that G contains an odd number of vertices of degree $(n-1)/2$.

2. Show that the Peterson graph cannot have a cycle of length 7.