Math 428
Graph Theory
Homework Set #2

Degree Sequences

1. Explain, using their formal definitions, why K_3 is not a subgraph of either the Peterson graph or the hypercube Q_n.

2. Let k and n be positive integers with $n > 2k$. Define $G_{n,k}$ to be the graph whose vertices are the k-element subsets of $\{1, 2, \ldots, n\}$ and where two vertices A and B are adjacent iff A and B are disjoint. Such graphs are called the *Kneser* graphs.

   (a) What is the order of $G_{n,k}$?

   (b) Is $G_{n,k}$ regular? If so, find this common degree.

3. Let u and v be adjacent vertices in G. Prove that uv is an edge in at least $\deg(u) + \deg(v) - n$ distinct triangles. (Hint: Consider the set $N(u) \cap N(v)$.)

4. Consider the following degree sequences. Determine if the sequence is graphical. If it is, give an example of such a graph. If it is not, explain why.

   (a) (4,3,3,2,2,1)

   (b) (6,3,3,3,3,2,2,1,1)

5. Consider the the following sequence

$$a, \ldots, a, (a-1), \ldots, (a-1)$$

   of n nonnegative integers whose sum is even and $a < n$. Show that this sequence is graphical.

6. Show that for every finite set S of nonnegative integers, there exists some integer k such that the sequence obtained by listing each element of S a total of k times is graphical. Find the minimum such k for $S = \{2, 6, 7\}$.

7. Prove that the sequence

$$1, 1, 2, 2, \ldots, k, k$$

   is graphical. (Hint: Use induction and consider whether k is even or odd.)

Optional Problems

1. Determine the order of the largest independent set in PG.