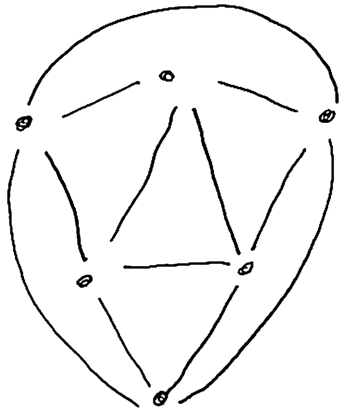
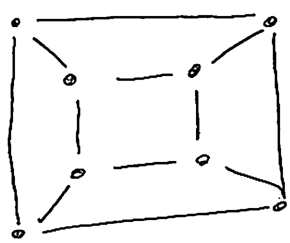
solutions HW #10

1)



2)

 $\cong Q_3$ (Hypercube)

This is bipartite as it has <u>no</u> odd cycle.

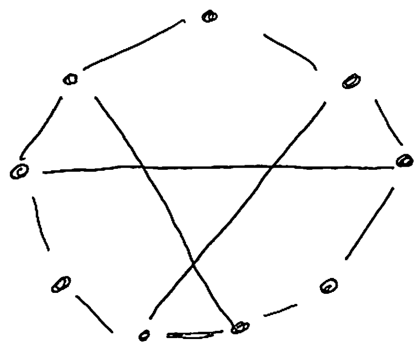
3) Assume not for a contradiction. Then $\exists$ a graph $G \ni$ every ~~deg~~ vertex v has $\deg v \geq 6$. By the HSL we have

$$2m = \sum_{v \in V(G)} \deg v \geq \underbrace{n}_{\text{order of } G} \cdot 6$$

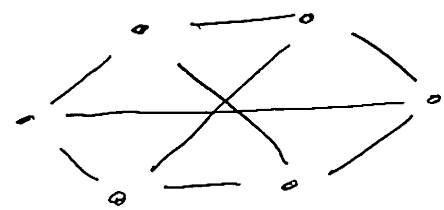
so $m \geq 3n > 3n-6$. But this contradicts thm 12.3.

4) As G is assumed to not have triangles, then for any region R, $\ell(R) \geq 4$. Using this information repeat the proof of thm 12.3.

5) A - Not planar:

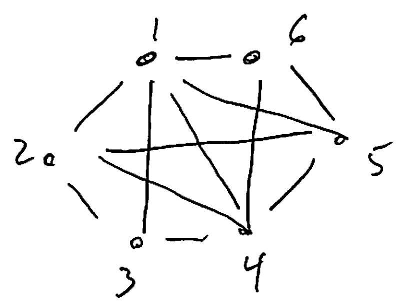


is a subdivision of

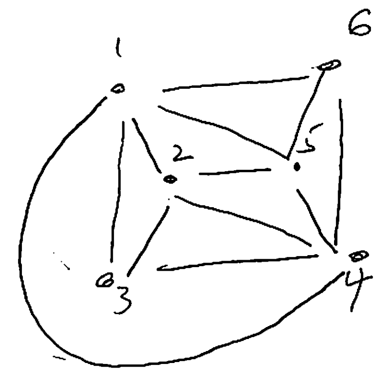


$K_{3,3}$

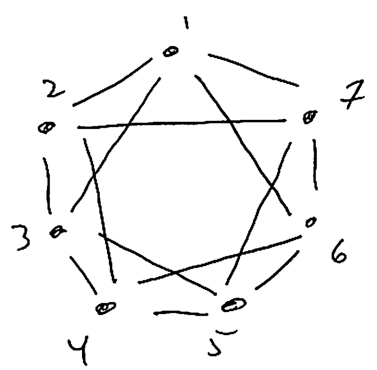
B - planar. It can be drawn as



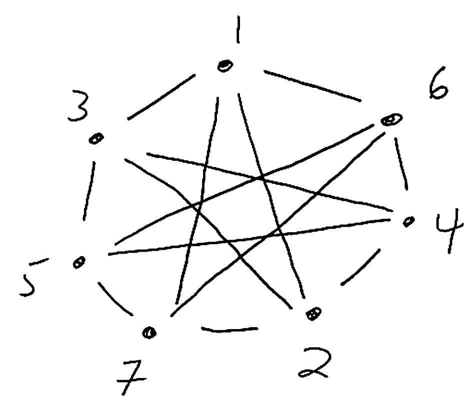
$\cong$



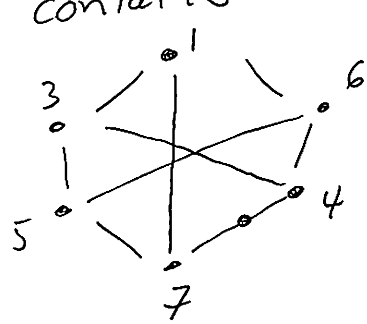
C - observe that



$\cong$



which contains



a subdivision of $K_{3,3}$.