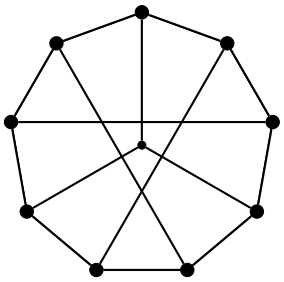
Math 428
Graph Theory
Homework Set #10

Planar Graphs

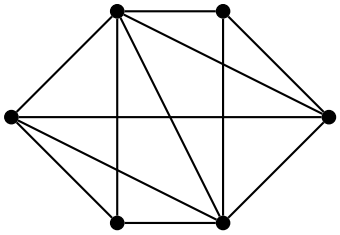
1. Find a 4-regular planar graph. (Note that K_5 is such a graph but it's not planar.)
2. Find a planar *bipartite* graph such that every vertex has degree 3. (Note that $K_{3,3}$ is such a graph but it's not planar.)
3. Show that every planar graph contains a vertex of degree at most 5. (Hint: Consider Theorem 12.3.)
4. In class we used the theorem

   Theorem (12.4)**.** *Let G have order n and size m. If it is connected and planar without triangles, then $m \leq 2n - 4$.*

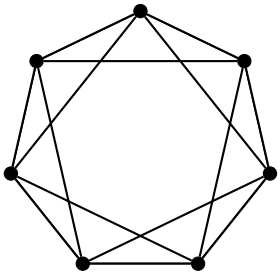
   to prove that $K_{3,3}$ is not planar. Prove this theorem.
5. Determine if the following three graphs are planar.



(A)



(B)



(C)

Note: The graph labeled (A) is isomorphic to the Peterson graph. Is the peterson graph planar?