Math 428

Graph Theory

Eulerian Graphs

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1. Give an example of a graph G such that G has an Eulerian trail but, for some edge e, $G - e$ is Eulerian.

2. Suppose that G is an r-regular graph of order n such that G and $\overline{G}$ are both connected. Is it possible that neither G nor $\overline{G}$ is Eulerian?

3. Let G and H be non-Eulerian connected regular graphs. Let F be the graph obtained by adding a new vertex x and joining x to each vertex in $G + H$. Show that F is Eulerian.

4. Show that if T is a tree containing at least one vertex of degree 2, then $\overline{T}$ is not Eulerian.

5. Let W be a closed walk in a graph G. Let H be the subgraph consisting of all the edges that appear an odd number of times in W. Prove that for each vertex $v \in V(H)$, $\deg_H(v)$ is even.