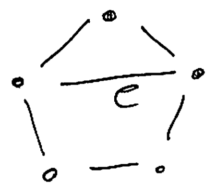
Eulerian Practice : Solutions

1) Consider

G: 

Then G has an Eulerian trail as it has exactly two odd deg, but $G-e$ is Eulerian.
(In fact any chorded cycle will work.)

2) Observe that if G is r-regular w/ order n then $\overline{G}$ is $(n-1-r)$-regular.

If G is not Eulerian then r must be odd. (Remember r is the common deg.) Since n must be even in this case (why?) then

$$n-1-r$$

is even. Thus $\overline{G}$ has all even deg so it is Eulerian.

To conclude, no, it is not possible for G & $\overline{G}$ to both be non-Eulerian.

3) If G & H are cntd non-Euleria graphs then as G is regular its common deg r must be odd. and as H is regular its common deg s is also odd. As in #2 $|V(H)|$ and $|V(G)|$ must be even. It follows by the construction of F that the new vertex has even deg $|V(H)| + |V(G)|$ and every other vertex in F also has even deg. $\therefore$ F is Eulerian.

4) If T has at least one verty of deg 2, then T cannot be K_1 in which case T has a leaf, a vertex of odd deg.

$\therefore$ T is not Eulerian.

5) Fix vertex x in H and let $\Sigma = \{e_1, \dots, e_r\}$ be all the edgs in H incident to x. Let a_i be the # of times edge e_i is walked in W. By assumption a_i is odd. Now, since W is closed every time we walk an edge e_i to x we must then walk some other edge e_j (i could equal j) away from x. Thus the total # of times an edge incident to x is walked is even. Thus

$$a_1 + \dots + a_r$$

is even. As all the a_i's are odd we must have r even. Thus $\deg x = |\Sigma| = r$ is even.