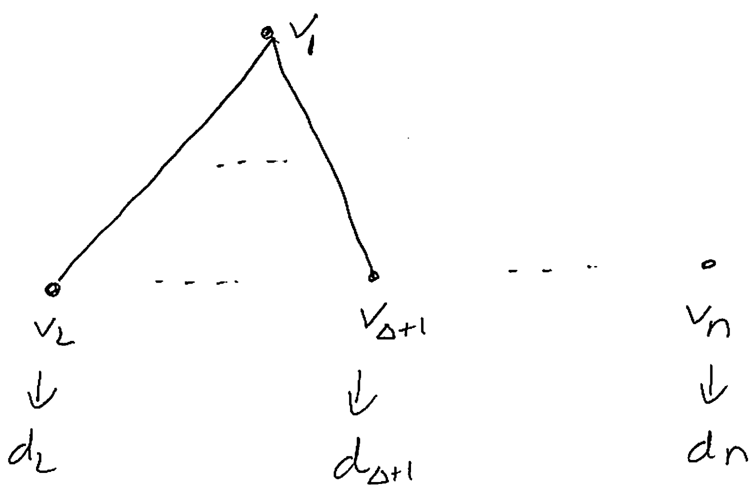
Lemma: Assume the seq

$$\Delta = d_1 \geq d_2 \geq \dots \geq d_n$$

is graphical. Then $\exists$ a graph G w/ $V(G) = \{v_1 \dots v_n\}$ $\ni$ $\deg v_i = d_i$ and $N(v_1) = \{v_2 \dots v_{\Delta+1}\}$.

In other words G looks like:

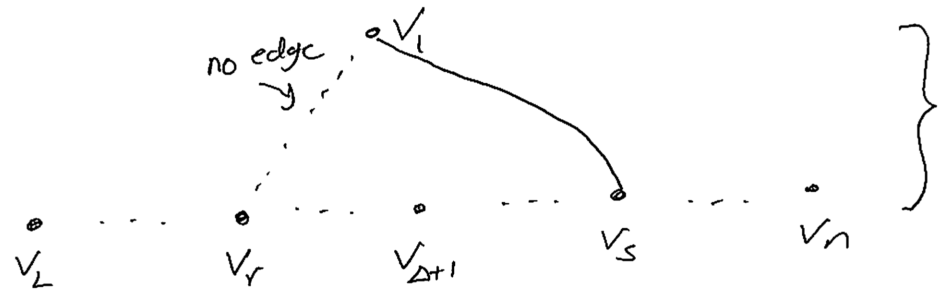


Pf: Assume for a contradiction that no such graph exists. Fix a vertex set $\{v_1 \dots v_n\}$ and consider all graphs on V $\ni$ $\deg v_i = d_i$.
Out of all these graphs choose G $\ni$ v_1 has the most # of n'bors in $\{v_2 \dots v_{\Delta+1}\}$.

Observe: As v_1 has Δ n'bors and (by assumption) they are not all in $\{v_2 \dots v_{\Delta+1}\}$ then

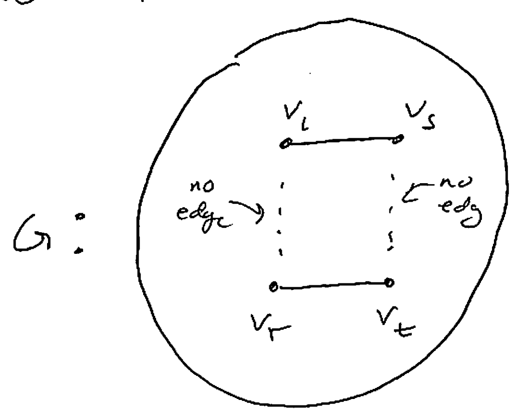
1. v_1 must be adj to some vertex outside this set
2. there must be some vertex in this set not adj to v_1.

so we get the following picture:

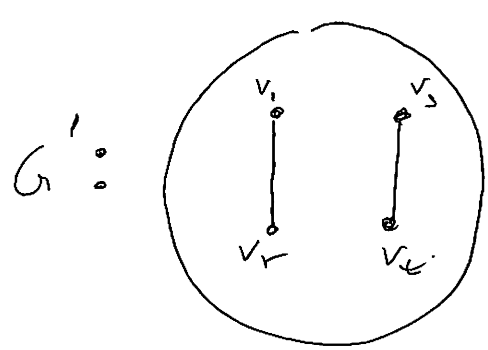


As our degree seq is nonincreasing $d_r \geq d_s$.
If we ignore the edge $v_1 v_s$ then v_s has $d_s - 1$ other edges. As $d_r > d_s - 1$ this means there is a vertex v_t .∋. $v_r \sim v_t \not\sim v_s$.

Now let us redraw G as:



Now "switch" these edges (and leave everything else the same to obtain



The new graph still has the deg seq (1), but in G' v_1 has one more n'bor in $\{v_2 \ldots v_{\Delta+1}\}$ (namely v_r) than it did in G. This contradicts our choice of G.