Math 350
Abstract Linear Algebra
Homework Set #2

Linear Dependence & Independence

1. Prove Lemma 2.1 from the notes.

2. Prove that if $\{v_1, \ldots, v_n\}$ spans V then so does

$$\{v_1 - v_2, v_2 - v_3, \ldots, v_{n-1} - v_n, v_n\}$$

3. (a) Let u, v be distinct vectors in V. Prove that $\{u, v\}$ is linearly dependent iff u is a multiple of v.

   (b) Give an example of three linearly dependent vectors in $\mathbb{R}^3$ such that no two are multiples of each other.

4. Prove Lemma 2.4 from the notes.

5. Let $p_0, \ldots, p_n \in \mathcal{P}_{\leq n}(\mathbb{R})$ such that $p_i(0) = 0$ for all i. Prove that these $n+1$ polynomials are not linearly independent.

6. Assume V is an $2n - 1$ dimensional vectors space. Prove that if U and W are n dimensional subspaces of V, then $U \cap W \neq \{0_V\}$.

7. Let V be a vector space having dimension n. Assume $S \subseteq V$ spans all of V. Prove that there is a subset of S which is a basis for V. (Note: This is not the same as the Basis Reduction Theorem. Here we do not know that S is finite.)