ULTRA Principles of Teaching

The following principles are not intended to capture all aspects of mathematics teaching. Rather, these are intended to make explicit particular aspects of mathematics teaching that are directly related to the ULTRA tasks and course aims. (ULTRA stands for Upgrading Learning for Teachers in Real Analysis.) As such, these are explicit learning goals of the ULTRA course.

**Principle of Teaching 1: Be purposeful about the definitions, arguments, or explanations used with students, acknowledging and revisiting mathematical limitations**

Teachers must explain mathematical ideas to students.

a) Good teaching: i) uses definitions that avoid introducing unnecessary mathematical limitations and yet are still accessible for the instructional context/audience; and ii) revisits limited definitions, pointing out and exemplifying the limitations to students

b) Good teaching: i) revisits limited arguments or explanations, pointing out and exemplifying the mathematical limitations to students; and ii) pushes students toward increasingly complete arguments or explanations

**Principle of Teaching 2: Clarify implicit assumptions and mathematical limitations in students’ mathematical statements or arguments**

Teachers must listen to students as they communicate about mathematical ideas.

a) Good teaching: i) analyzes students’ mathematical statements or arguments and recognizes when they rely on implicit assumptions; and ii) determines when (not if) the appropriate moment would be at which to push/probe the student in their thinking, in particular making explicit the implicit assumption.

b) Good teaching: i) analyzes the generality of students’ statements, argument, processes, and explanations; and ii) makes explicit to the student and/or class when a student’s statement, argument, process, or explanation is or is not valid

**Principle of Teaching 3: Use graphical (visual) approaches when productive for student learning**

Teachers must represent mathematical ideas to students.

a) Good teaching uses graphical (visual) approaches when this would help students develop deeper insight into and learn about a definition, statement, argument, proof, process, explanation, or concept.

**Principle of Teaching 4: Select examples that exemplify nuances within and boundaries around a mathematical idea**

Teachers must use examples to exemplify mathematical ideas to students.

a) Good teaching strategically selects a set of examples for a definition, statement, argument, proof, or process that has sufficient breadth to support rich concept development and to exemplify each of the nuances within and the boundaries around the mathematical idea.
Principle of Teaching 5: Clarify the logic underlying students’ mathematical statements, arguments, proofs or processes

Teachers must listen to students as they communicate about mathematical ideas.

a) Good teaching clarifies the logic underlying students’ mathematical statements, arguments, proofs, or processes when this would help make explicit why a particular statement, argument, proof, or process is or is not valid.

Principle of Teaching 6: Design instructional sequences and tasks that draw on mathematical developments and relationships

Teachers must develop instructional sequences for mathematical ideas.

a) Good teaching includes designing instructional sequences and tasks that make clear and explicit the transition from a mathematical idea in relation to one set of objects as it grows to consider broader sets of objects, in particular clarifying both the rationale as well as the nuanced differences that arise.
b) Good teaching includes using the mathematical practice of modeling more complex ideas with simpler (easier to understand) ideas as a pedagogical practice in their instruction, specifically structuring instructional sequences and tasks that build on what students currently know about simpler ideas to help them understand more complex ideas.

Principle of Teaching 7: Avoid giving rules without the accompanying mathematical explanation

Teachers must explain mathematical ideas to students.

a) Good teaching provides students with an opportunity to understand the mathematical reasons for why certain rules, procedures, and processes work, in a manner still accessible for the instructional context/audience.

Principle of Teaching 8: Use different types of explanations to help students understand a mathematical idea

Teachers must engage individual students as they explain mathematical ideas.

a) Good teaching: i) recognizes that individual students have different prior backgrounds in and knowledge of mathematics, which, in turn, makes some mathematical explanations or techniques more appealing and understandable; and ii) thus, incorporates different types of explanations (or representations, etc.) to help students understand a particular mathematical idea.