Research Statement

My research as a graduate student at Rutgers University has been in discrete math. The bulk of my research has been on an extension of the stable matching problem, though I have also done some work on sensitivity measures of Boolean functions.

The stable matching problem is a compelling problem that borders graph theory, game theory, and combinatorial algorithms; in it, we consider a system I of n men and n women, where each individual has a preference ordering of the individuals of the opposite gender. (We generally consider this system to be on a graph G_I, where the vertex set is the set of all individuals, and the edge set is every possible man-woman pair.) A matching is unstable over an edge if both vertices of the edge prefer each other to their respective partners in the matching, and is stable if it is not unstable over any edge. The applications of this problem are numerous – one well-documented example is the task of assigning recently graduated doctors to residencies. The famous 1962 paper by David Gale and Lloyd Shapley showed that a stable matching always exists, and an algorithm exists to construct one in $O(n^3)$ time – the resulting Gale-Shapley algorithm is used by the NRMP for assigning residencies.

My research introduces a relaxation of the usual condition of stability that I called S-stability, where S consists of some subset of the edges of G_I; a matching is S-stable if it is not unstable over any edge in S. We can consider the S-stable matchings to be the matchings that no pair objects to if all agents agree that the edges outside of S are "superfluous" and not worth worrying about for instabilities. One interesting aspect of my research involves the function Ψ, which maps S to the set of all edges that appear in an S-stable matching. A specific question that I considered was whether there exists an S such that $\Psi(S) = S$ for any instance of the stable matching problem, and if such a set of edges is unique; if so, we define the S-stable matchings for such an S as pseudostable. In particular, a fixed point of Ψ would represent a set of edges S where the superfluous edges are precisely those that don't appear in any S-stable matching.

My investigations into the questions that arise from such a system have yielded some interesting results. While the general problem of finding the set of S-stable matchings (and thereby $\Psi(S)$) is very difficult for general S, I have found specific conditions under which we can find a simple algorithm to compute it. I have found that a fixed point under Ψ always exists and is unique, and devised an algorithm that constructs it in $O(n^3)$ time. Furthermore, the pseudostable matchings can be arranged in a distributive lattice under the same ordering as the lattice of stable matchings. I have also shown that, under specific ways of truncating the preference lists of the vertices of G_I, the behavior of Ψ on the remaining edges is preserved.

I have also done some work on sensitivity measures of Boolean functions, a subfield recently brought to public attention by the proof of the sensitivity conjecture. One such measure is the degree of a Boolean function $f: \{0,1\}^n \to \{0,1\}$, the minimum degree of a polynomial $f^*: \mathbb{R}^n \to \mathbb{R}$ that matches f over $\{0,1\}^n$. Previously, it has been known that the maximum number of variables a degree d Boolean function has is between 2^d and $d2^{d-1}$. I proved that every such Boolean function has at most $C * 2^d$ variables for some universal constant C, tightening the possible range of this maximum to a constant factor. The proof of this can be seen in the paper, "An Asymptotically Tight Bound on the Number of Relevant Variables in a Bounded Degree Boolean Function," co-written with my advisor, Michael Saks, and Pooya Hatami, then a postdoc at Rutgers University.

Applications to Undergraduate Teaching

I believe that my field of choice gives me a unique perspective on the subject of undergraduate teaching. More students than ever are looking to go into computer science and engineering, and the problems in discrete math – for example, the algorithmic problems connected to the stable matching problem that I have worked with – are highly relevant to students studying computers. As an example of this correlation, at Rutgers, combinatorics and graph theory consistently have high enrollment, with a large percentage of the students taking them being computer science majors. I have also taught graph theory at Rutgers as the lecturer of record.

Furthermore, familiarity with discrete math gives me the ability to introduce students to a greater paradigm of approaching mathematics. Many students think of math in terms of numbers and equations and formulas; this conception is often reinforced by the track of algebra into calculus that many students go through, and the plethora of evaluation problems that they do along the way. They see it as a dry exercise in applying memorized rules, and purely a prerequisite to the courses they want to take. Discrete math is particularly useful for challenging that notion, since it breaks away from the methodologies of calculus and looks at very different types of systems. Learning about discrete systems is where the principle that mathematics is a creative process is laid bare in an accessible way, and demonstrates that an open mind and an eye for details are the most valuable tools in a mathematician's toolkit – a lesson that is valuable for every field and application of mathematics.