Asymptotic notation

Notation below is used for f, g functions of some parameter, e.g. n or x, which (as here) the notation often suppresses. The limiting statements are meant as the parameter approaches some limit (most often $n \to \infty$); the others are for the parameter in some specified range.

$f \sim g$: $f/g \to 1$
$f = O(g)$: $|f|/|g|$ is bounded above
$f = o(g)$: $f/g \to 0$ (also written $f \ll g$)
$f = \Omega(g)$: $g = O(f)$
(equiv: $|f|/|g|$ is bounded below by a positive constant)
$f = \omega(g)$: $|f|/|g| \to \infty$ (equiv: $g = o(f)$)
$f = \Theta(g)$: $f = O(g)$ *and* $g = O(f)$ (also written $f \asymp g$)
(equiv: $|f|/|g|$ lies between two positive constants)
$f \lesssim g$: $\limsup f/g \leq 1$ (not sure we'll see this one)

We can then, for example, write simply $O(g)$ to mean any (perhaps unspecified) function whose absolute value is known to be bounded above by Cg for some fixed C. Big and little "Oh" are often used for error terms, for example

$$e^x = 1 + x + O(x^2) \quad \text{as} \quad x \to 0,$$

in which case the functions $O(\cdot)$, $o(\cdot)$ will often be negative. In most (or all?) of our other uses f and g will be positive.