Quiz #1

1. Find an equation of the line whose slope is \(-3\) and which passes through the point \((1, 4)\).

2. Simplify the expression \(\frac{x^3 - 4x}{x^3 - x^2 - 6x}\) as much as possible.

3. Write the expression \(\frac{\sqrt{xy^3}}{(x^2/3y^{-5/2})^6}\) in the form \(x^a y^b\).

4. Let \(f(x) = 3x^2\). Simplify the expression \(\frac{f(x + h) - f(x)}{h}\) as much as possible.

5. Evaluate the expression \(\log_6(9) + \log_6(4)\).

6. Let \(f(x) = 5x + 1\). Evaluate \(f^{-1}(1)\).

7. Write the solution to the inequality \(x^2 - 3x + 2 < 0\) using interval notation.

8. Find all values of \(\theta\) in the interval \([0, 2\pi)\) such that \(2\sin(2\theta) = 1\).

Quiz #2

9. Find an equation of the line that passes through the point \((-\pi, 1)\) with slope \(\sqrt{2}\).

10. Find the center and radius of the circle described by the equation \(x^2 - 6x + y^2 + 2y - 6 = 0\).

11. Let \(f(x) = \frac{1}{x}\). Simplify the difference quotient \(\frac{f(x + h) - f(x)}{h}\) as much as possible.

Quiz #3

12. Evaluate the limits using the given graph.

\[
\begin{align*}
(a) \quad \lim_{x \to -2^+} f(x) &= \quad \quad \quad \quad \quad \quad \quad (c) \quad \lim_{x \to 4^+} f(x) = \\
(b) \quad \lim_{x \to 4^-} f(x) &= \quad \quad \quad \quad \quad \quad \quad (d) \quad \lim_{x \to 6} f(x) =
\end{align*}
\]
13. Evaluate each of the following limits or show why it does not exist.

(a) \( \lim_{{x \to 2}} \left( \frac{2x^2 - 3x - 2}{x^2 + 2x - 8} \right) \) 
(b) \( \lim_{{x \to 4}} \left( \frac{3 - \sqrt{x+5}}{x - 4} \right) \)

Quiz #4

14. Consider the following function.

\[
f(x) = \begin{cases} 
  x^3 + 27 & , \quad x \leq -3 \\
  \frac{x + 3}{2 - \sqrt{1 - x}} & , \quad -3 < x < 1 \\
  4 & , \quad x = 1 \\
  x^2 + 2x - 1 & , \quad 1 < x 
\end{cases}
\]

(a) Find all points where \( f \) is discontinuous. Be sure to give a full justification here.
(b) For each \( x \)-value you found in part (a), determine what value should be assigned to \( f \), if any, to guarantee that \( f \) will be continuous there. Justify your answer.

(For example, if you claim \( f \) is discontinuous at \( x = a \), then you should determine the value that should be assigned to \( f(a) \), if any, to guarantee that \( f \) will be continuous at \( x = a \).)

15. Find all real solutions to the following equation.

\[ \log_2(x) + \log_2(x - 3) = 2 \]

Quiz #5

16. Let \( f(x) = \frac{3 - x}{1 + x} \). Use the limit definition of derivative to calculate \( f'(1) \).

If you simply quote a rule, you will receive zero credit. You must use the definition of derivative.

17. Let \( g(x) = x^2 \ln(x) \). Find an equation of the tangent line at \( x = e \).
18. At a certain factory, the total cost (in dollars) of manufacturing $q$ tables during the daily production run is

$$C(q) = 0.2q^2 + 10q + 900$$

From experience, it has been determined that approximately

$$q(t) = t^2 + 99t$$

tables are manufactured during the first $t$ hours of a production run.

Make sure to indicate the units of your answer in each question below.

3 pts (a) Calculate $C'(50)$ and explain its precise meaning.

3 pts (b) Compute the rate at which the total manufacturing cost is changing with respect to time one hour after production begins.

19. Calculate \( \frac{d}{dx} \left( 4x^3 e^{\sin(2x)} \right) \). After computing the derivative, do not simplify your answer.

Quiz #7

5 pts 20. Find an equation of the line tangent to the graph of

$$x^3 + y^3 = y + 21$$

at the point $(3, -2)$.

5 pts 21. A ladder 13 feet long rests against a vertical wall and is sliding down the wall at the rate of 3 feet per second at the instant the foot of the ladder is 5 feet from the base of the wall. At this instant, how fast is the foot of the ladder moving away from the wall?

You must include correct units as part of your answer.

Quiz #8

5 pts 22. Use a linear approximation to estimate the value of \( \frac{1}{\sqrt{0.96}} \).

You must express your answer as a single exact rational number.

5 pts 23. Find the absolute maximum and absolute minimum values of $f(x) = \frac{10x}{x^2 + 1}$ on the interval $[0, 2]$.

Quiz #9

11 pts 24. The function $f$ and its derivatives are given below.

$$f(x) = \frac{x}{x^2 + 1}, \quad f'(x) = \frac{1 - x^2}{(x^2 + 1)^2}, \quad f''(x) = \frac{2x(x^2 - 3)}{(x^2 + 1)^3}$$

(a) Fill in the table below with information about the graph of $y = f(x)$. Write “NONE” as your answer if appropriate.
domain of $f(x)$

vertical asymptotes

horizontal asymptotes

intervals where $f$ is decreasing

intervals where $f$ is increasing

$x$- and $y$-coordinates of local minima

$x$- and $y$-coordinates of local maxima

intervals where graph is concave down

intervals where graph is concave up

points of inflection

(b) Sketch the graph of $y = f(x)$ on the provided graph paper. Make sure to label the scales on the axes! For each relative extremum or inflection point, identify its coordinates and label the point “rel. min”, “rel. max”, or “infl. pt.” as appropriate.

### Quiz #10

#### 25. Calculate the following limit or show it does not exist. Show all work.

$$\lim_{x \to 0} \frac{x - \ln(1 + x)}{1 - \cos(2x)}$$

#### 26. The product of two positive numbers is 25. Find the smallest value of their sum.

### Quiz #11

#### 27. An apartment complex has 200 units. When the monthly rent for each unit is $1200, all units are occupied. Experience indicates that for each $40-increase in rent, 10 units will become vacant. Each rented apartment costs the owners of the complex $480 per month to maintain. What monthly rent should be charged to maximize the owner’s profit?

#### 28. Calculate the following antiderivatives.

(a) $\int (\cos(w) + 2 \sin(w) - 3e^w) \, dw$

(b) $\int \frac{3t^3 - \sqrt{t} + 2t}{t^2} \, dt$
Quiz #12

29. Let \( f(x) = x^2 + 3x \) and let \( R \) be the region under the graph of \( y = f(x) \) and above the interval \([0, 2]\) on the \( x \)-axis.

2 pts  
(a) Sketch the region \( R \).

4 pts  
(b) Estimate the area of \( R \) using a Riemann sum with right endpoints and 4 rectangles. 

Do not simplify your answer.

4 pts  
(c) Calculate the exact area of \( R \). 

Simplify your answer as much as possible.