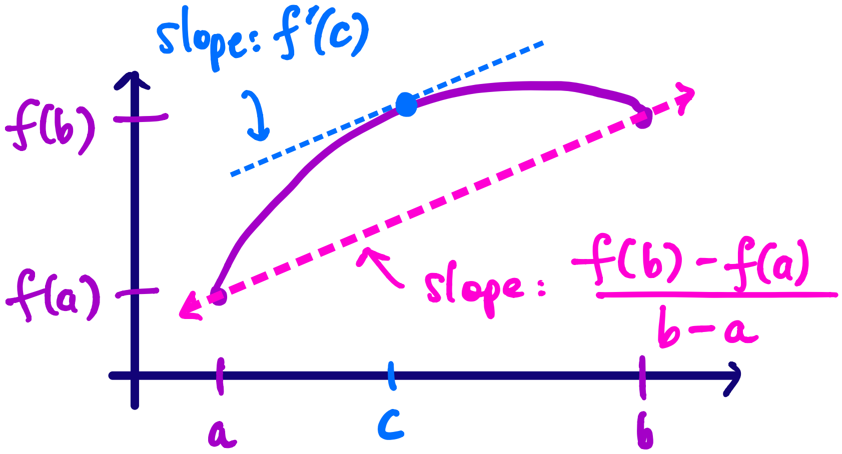
Optional: Mean Value Theorem

Thm: (MVT) Suppose f is continuous on $[a,b]$ and differentiable on (a,b). Then there exists c in (a,b) such that

$$\frac{f(b)-f(a)}{b-a} = f'(c)$$

Graphical interpretation of MVT:



MVT says there is a tangent line parallel to the secant line

Ex. 1

Let $f(x) = x^{2/3}$. For each interval, determine whether the hypotheses of the MVT are satisfied. If yes, find all values of c decribed by the MVT.

(a) $[-1, 1]$ (b) $[0, 8]$

Solution:

- Where is f continuous?

Power functions are continuous on their domain, so $f(x) = x^{2/3}$ is continuous on $(-\infty, \infty)$. So the continuity hypothesis of MVT is satisfied on all intervals.

• Where is f differentiable?

Since $0 < 2/3 < 1$, we know $f(x) = x^{2/3}$ is differentiable everywhere except $x = 0$. So any open interval with $x=0$ does not satisfy the MVT hypotheses.

(a) Since $x=0$ is in $(-1,1)$, the MVT hypotheses are not satisfied.

(b) Since $x=0$ is <u>not</u> in $(0,8)$, the MVT hypotheses are satisfied. Hence there is some c in $(0,8)$ with:

$$f'(c) = \frac{f(8) - f(0)}{8-0}$$

$$\frac{2}{3}c^{-1/3} = \frac{8^{2/3} - 0}{8-0} = 8^{-1/3}$$

Solving for c gives

$$c = \left(\frac{3}{2}\right)^{-3} \cdot 8 = \frac{64}{27}$$

Important special case of MVT:

<u>Thm</u>: (Rolle's Theorem) Suppose f is continuous on $[a,b]$, differentiable on (a,b), and $f(a) = f(b)$. Then there exists c in (a,b) with $f'(c) = 0$.
