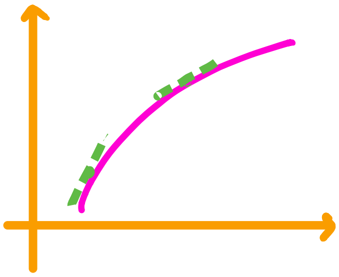
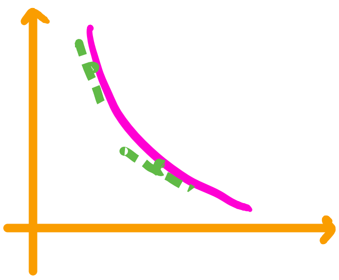
Section 4.3 Supplement: Conceptual Background

First Derivative $f'(x)$

What f' says about the graph of f:

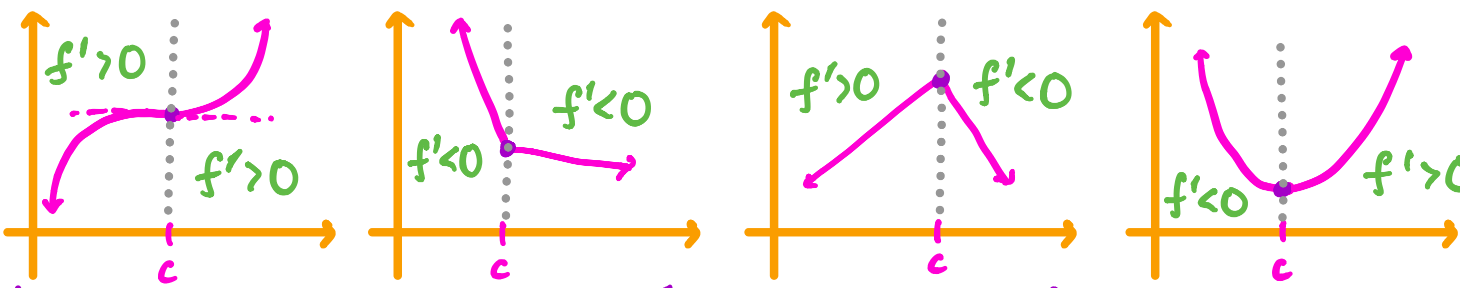


- $f'(x) > 0$ (tangents have positive slope)
- f is increasing (as x increases, y increases)



- $f'(x) < 0$ (tangents have negative slope)
- f is decreasing (as x increases, y decreases)

What about local extreme values? Suppose $x = c$ is a critical point of f. (So $f'(c) = 0$ or $f'(c)$ dne.)



- f' does <u>not</u> change sign at $x = c$
- f has no local extremum at $x = c$

- f' changes from ⊕ to ⊖ at $x = c$
- local max

- f' changes from ⊖ to ⊕ at $x = c$
- local min

Summary of information from $f'(x)$:

Sign of $f'(x)$ on (a, b)	Shape of $f(x)$ on (a, b)
⊖	decreasing
⊕	increasing

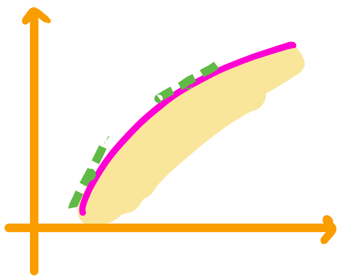
Sign change of f' at $x=c$	Classification of $f(c)$
$-$ to $+$	local minimum
$+$ to $-$	local maximum
no change	not a local extremum

* Assume $x=c$ is a critical point ($f'(c)=0$ or $f'(c)$ dne)

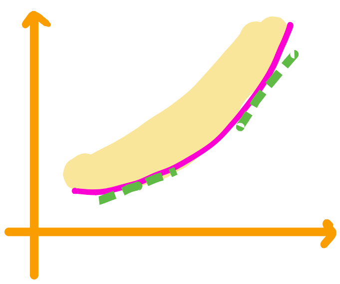
Second Derivative $f''(x)$

What f'' says about the graph of f:

Note: $f'(x)>0$ in both graphs but how is $f'(x)$ changing?

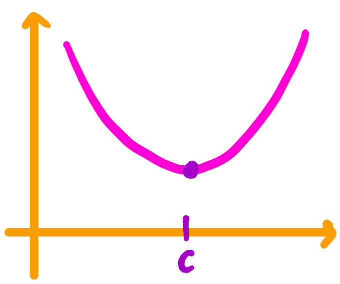


- $f'(x)$ is decreasing (slope gets less positive)
- $f''(x)<0$ (concave down)
- graph of f is below tangent lines

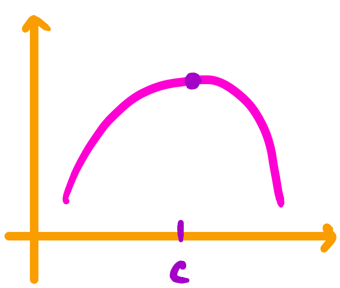


- $f'(x)$ is increasing (slope gets more positive)
- $f''(x)>0$ (concave up)
- graph of f is above tangent lines

What about local extreme values? Suppose $x=c$ is a critical point of f and f'' is continuous at $x=c$.



- $f'(c)=0$
- $f''(c)>0$
- $f(c)$ is a local minimum



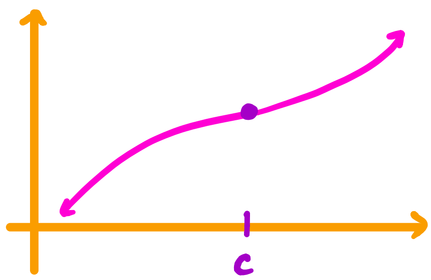
- $f'(c)=0$
- $f''(c)<0$
- $f(c)$ is a local maximum

Summary of information from $f''(x)$:

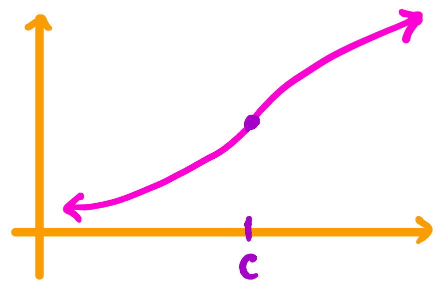
Sign of $f''(x)$ on (a,b)	Shape of $f(x)$ on (a,b)
$-$	concave down
$+$	concave up

$f'(c)=0$, sign of $f''(c)$	Classification of $f(c)$
$-$	local maximum
$+$	local minimum
zero	unknown

* Inflection points occur where $f(x)$ is continuous and $f''(x)$ changes sign



inflection from concave down to concave up



inflection from concave up to concave down

Graphing $y=f(x)$

① Info from $f(x)$:
- points on graph
- vertical asymptotes
- horizontal asymptotes

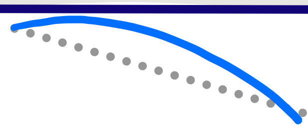
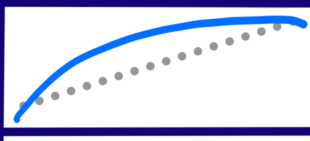
② Info from $f'(x)$:
- find where $f'(x)=0$ or where $f'(x)$ dne
- construct sign chart for $f'(x)$
- infer intervals of increase/decrease
- determine local extrema

③ Info from $f''(x)$:

- find where $f''(x) = 0$ or where $f''(x)$ dne
- construct sign chart for $f''(x)$
- infer intervals of concavity
- determine inflection points
- (optional: verify local extrema)

④ Graph $y = f(x)$:

- list important points (local extrema, inflection pts, etc.)
- summarize all info from f, f', and f''
- use chart below to sketch graph:

increase / concavity	decreasing	increasing
concave down		
concave up