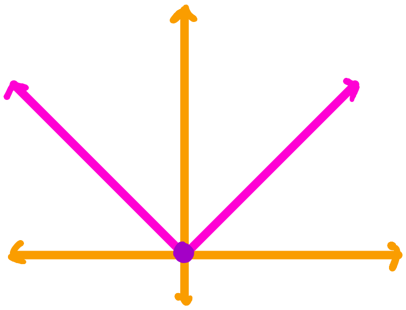
Section 4.1 Supplement: Catalog of Non-differentiable Functions

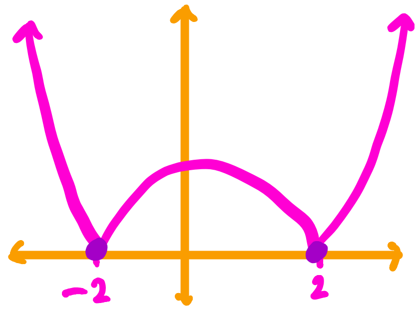
In Section 3.1, we learned how to recognize non-differentiable functions graphically. The notes below describle how to recognize analytically functions that are continuous but not differentiable. There are three main categories:

① Absolute Value: $f(x) = |x|$

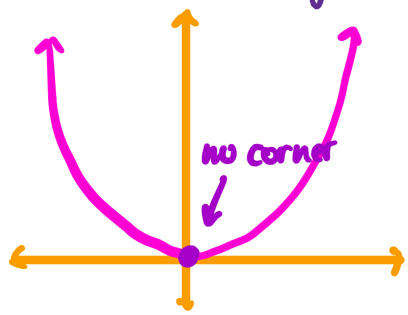
The function f is not differentiable at $x=0$. The graph of f has a sharp corner at $x=0$. So the function $h(x) = |g(x)|$ is (possibly) not differentiable where $g(x)=0$.



$y = |x|$



$y = |x^2-4|$

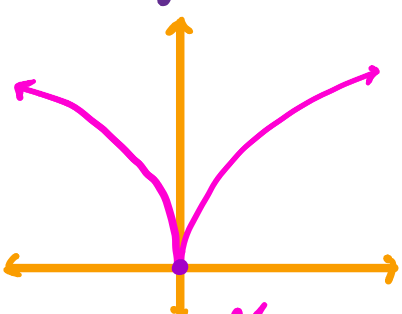


$y = |x^3|$

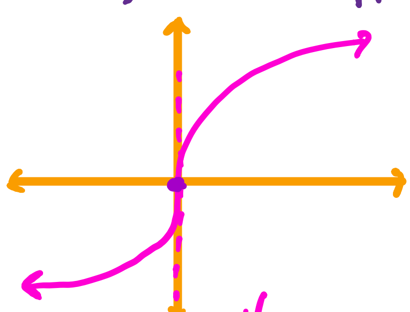
Ex: The function $h(x) = |x^2-4|$ is not differentiable at $x=-2$ and $x=2$. ($x^2-4=0 \iff x = \pm 2$)

② Power Functions: $f(x) = x^n$ $(0 < n < 1)$

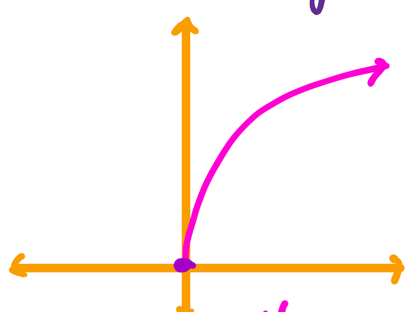
The function f is not differentiable at $x=0$. The graph has a cusp or vertical tangent at $x=0$. So the function $h(x) = g(x)^n$ is (possibly) not differentiable where $g(x)=0$



$y = x^{2/3}$

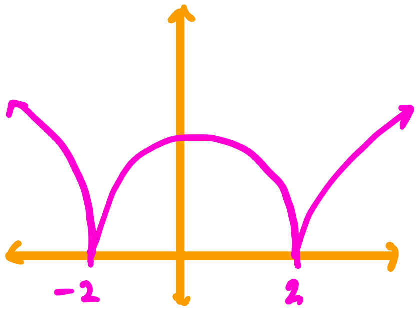


$y = x^{1/3}$

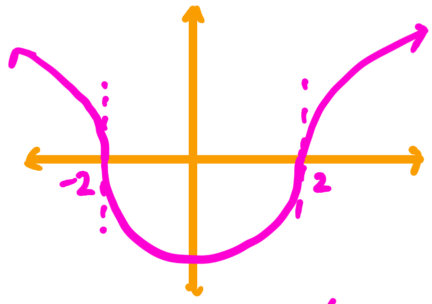


$y = x^{1/2}$

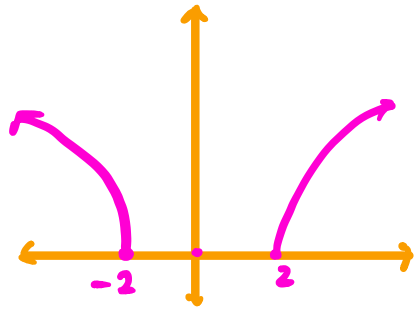
Ex: The function $h(x) = (x^2-4)^{2/3}$ is not differentiable at $x=-2$ and $x=2$. ($x^2-4=0 \Leftrightarrow x = \pm 2$). Similarly, both $F(x) = (x^2-4)^{1/3}$ and $G(x) = (x^2-4)^{1/2}$ are not differentiable at $x=-2$ and $x=2$.



$y=(x^2-4)^{2/3}$



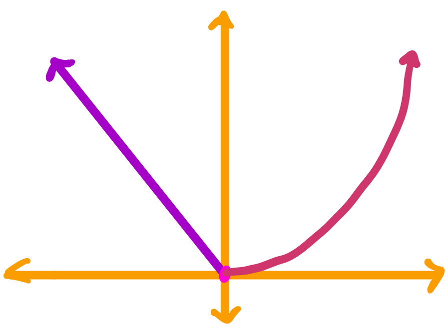
$y=(x^2-4)^{1/3}$



$y=(x^2-4)^{1/2}$

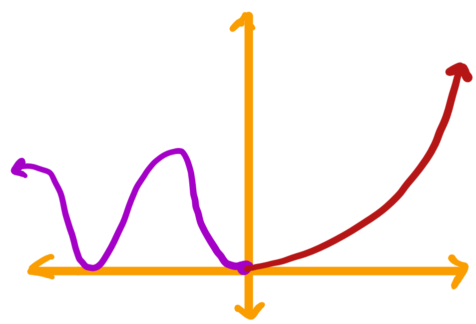
③ Piecewise-defined Functions

Very often (but not always), a piecewise-defined function is not differentiable at the transition points.



$$y=\begin{cases} -x & \text{if } x<0 \\ x^2 & \text{if } x\geq 0 \end{cases}$$

not differentiable at $x=0$



$$y=\begin{cases} 1-\cos(x) & \text{if } x<0 \\ x^2 & \text{if } x\geq 0 \end{cases}$$

differentiable at $x=0$