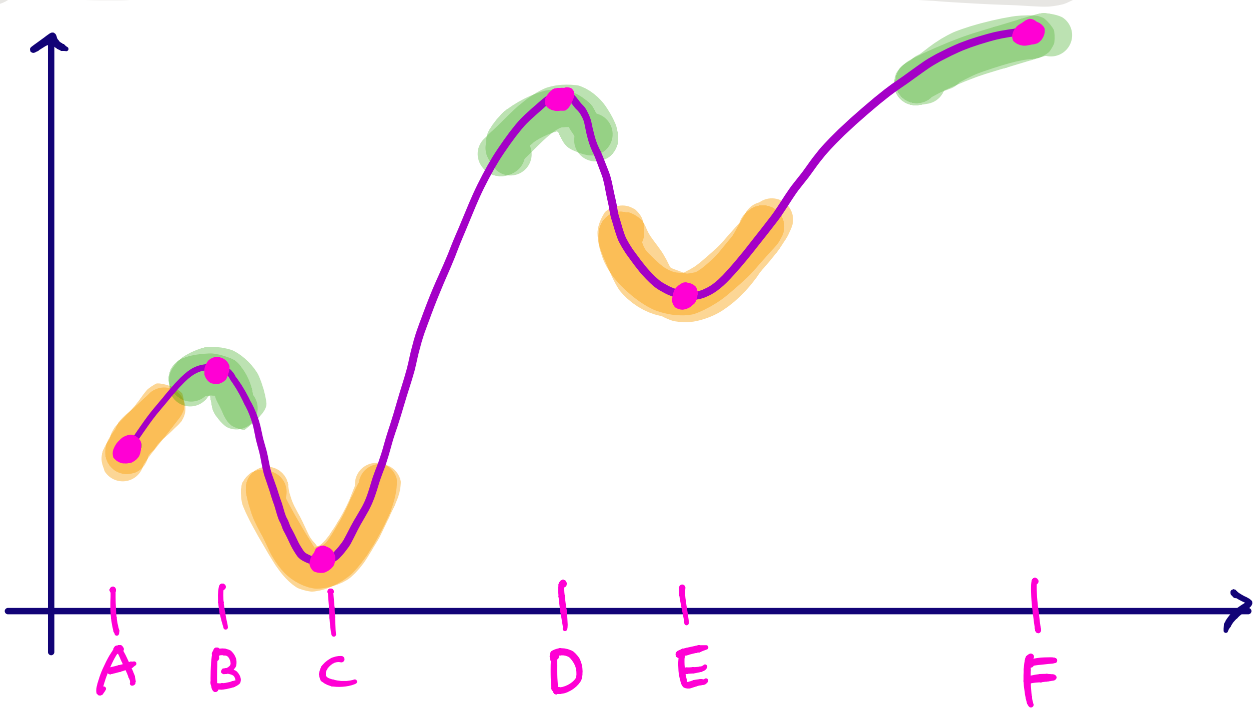
Section 4.1 Supplement: Conceptual Background

Basic Definitions:

Absolute minimum value of f on $[a,b]$	Relative minimum value of f at $x=c$
If $f(c)$ is the abs. min. value of f on $[a,b]$, then $f(c)$ is the least possible value of f for all x in $[a,b]$.	If $f(c)$ is a relative min. value of f, then $f(c)$ is the least possible value of f for x near c.

- similar definitions for absolute maximum and relative maximum (replace "least" with "greatest")
- "global" = "absolute" and "local" = "relative"
- "extremum" means minimum or maximum

Locating local extreme values graphically



Relative Minimum Values : ~~f(A)~~, f(C), f(E)

your textbook does not allow relative extrema at boundary points

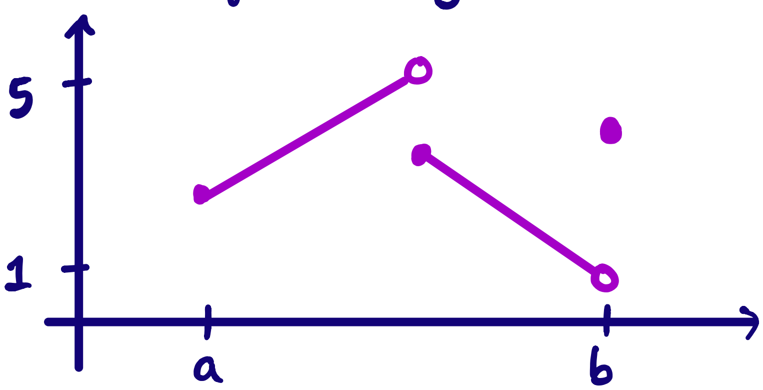
also absolute minimum

Relative Maximum Values: f(B), f(D), ~~f(F)~~

your textbook does allow absolute extrema at boundary points

Thm: (Extreme Value Theorem, EVT)
Suppose f is continuous on the closed, bounded interval $[a,b]$. Then the absolute min. and max. of f on $[a,b]$ exist.

What can go wrong if f is not continuous?

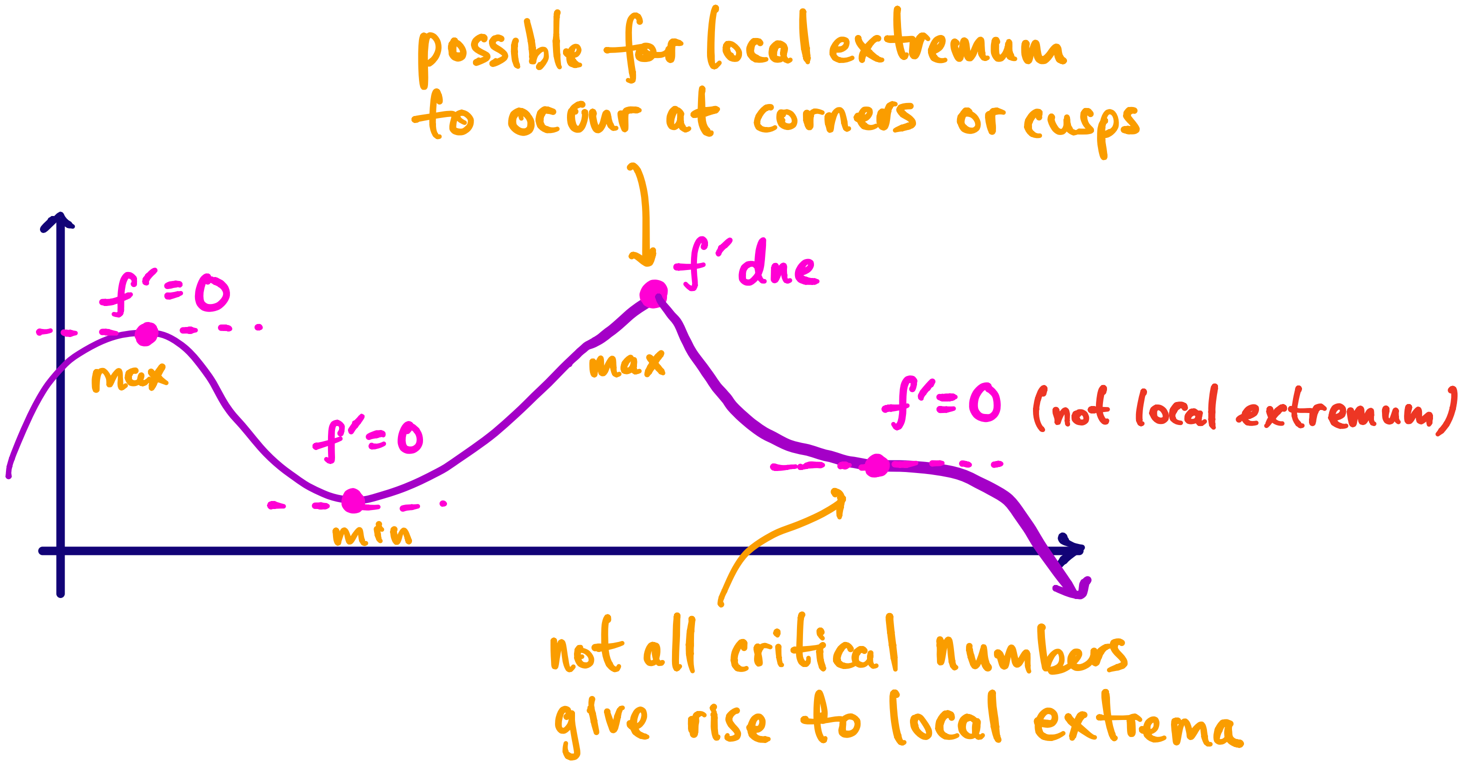


domain: $[a,b]$
no absolute min.
no absolute max.

Def: A number c in the interior of the domain of f is a critical number if $f'(c)$ DNE or $f'(c)=0$.

Thm: (Fermat)
If $f(c)$ is a local extremum, then c is a critical number.

* Why do we need to check where $f'(x)$ dne?



Algorithm for finding absolute extrema of f on $[a,b]$

① Is f continuous on $[a,b]$?

- If no, stop. (EVT concludes nothing.)
- If yes, continue to ②.

② Find critical numbers of f in (a,b).

- find where $f'(x)$ dne
- solve the equation $f'(x)=0$

③ Make a table of <u>candidate</u> extreme values of $f(x)$:

- values of f at critical numbers
- values of f at endpoints ($x=a$ and $x=b$

④ Least candidate value is absolute min.
Greatest Candidate value is absolute max.

* Discard other values

* Okay for abs. extremum to occur at more than one x-value.