“Paradoxes” in Game Theory

Or, Why I’m Not an Economist

Jake Baron

What is Game Theory?
(first attempt)

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- Analysis of “strategic situations”

What is Game Theory? (philosophically)

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- Three views:
  - Descriptive
  - Proscriptive
  - Pure mathematics

What is Game Theory? (mathematically)

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- Optimization over a Cartesian product

What is a game (mathematically)?
[i.e. it's time for some real math now]

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- Record all relevant information in a tuple

$$(P, \ (S_i)_{i \in P}, \ (u_i)_{i \in P})$$

P is a nonempty set of ***players***

S_i is a nonempty set of ***strategies*** for player i

$u_i: \Pi_{j \in P} S_j \rightarrow \mathbf{R}$ is a ***utility function*** for player i

Traveler's Dilemma
(Kaushik Basu, 1994)

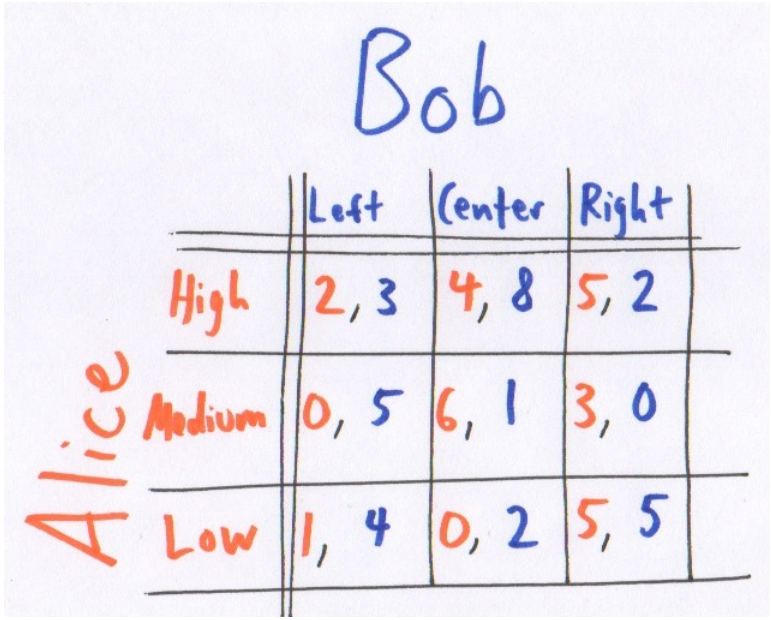
What should I say my urn is worth?
($2 to $100, integer number of $)

Bob

Alice

	$100	$99	$98	$97	...	$3	$2
$100	100, 100	97, 101	96, 100	95, 99	...	1, 5	0, 4
$99	101, 97	99, 99	96, 100	95, 99	...	1, 5	0, 4
$98	100, 96	100, 96	98, 98	95, 99	...	1, 5	0, 4
$97	99, 95	99, 95	99, 95	97, 97	...	1, 5	0, 4
⋮	⋮	⋮	⋮	⋮	⋱	⋮	⋮
$3	5, 1	5, 1	5, 1	5, 1	...	3, 3	0, 4
$2	4, 0	4, 0	4, 0	4, 0	...	4, 0	2, 2

Here's a simpler game



Here's a simpler game

		Bob		
		Left	Center	Right
Alice	High	2, 3	4, 8	5, 2
	Medium	0, 5	6, 1	3, 0
	Low	1, 4	0, 2	5, 5

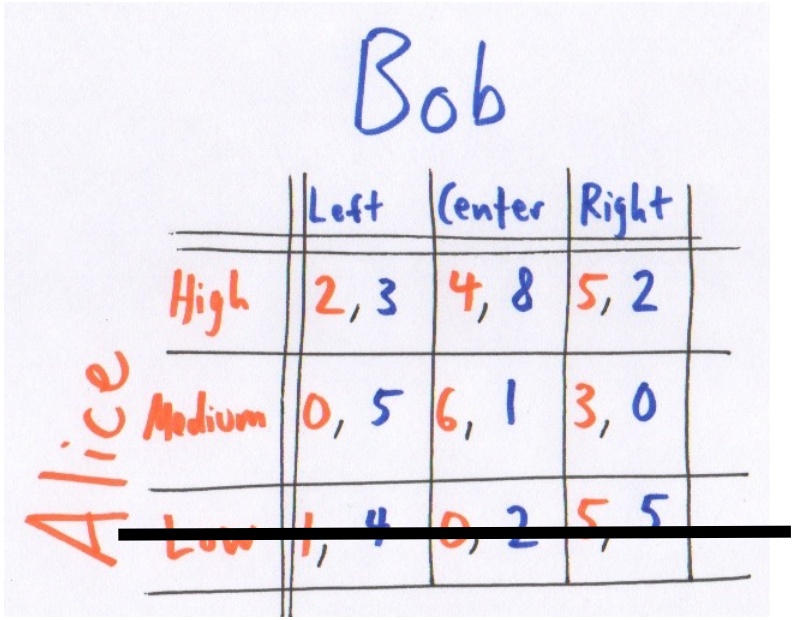
Let's play!

Here's a simpler game

Alice \ Bob	Left	Center	Right
High	2, 3	4, 8	5, 2
Medium	0, 5	6, 1	3, 0
Low	1, 4	0, 2	5, 5

- For Alice, Low is ***dominated*** by High

Here's a simpler game



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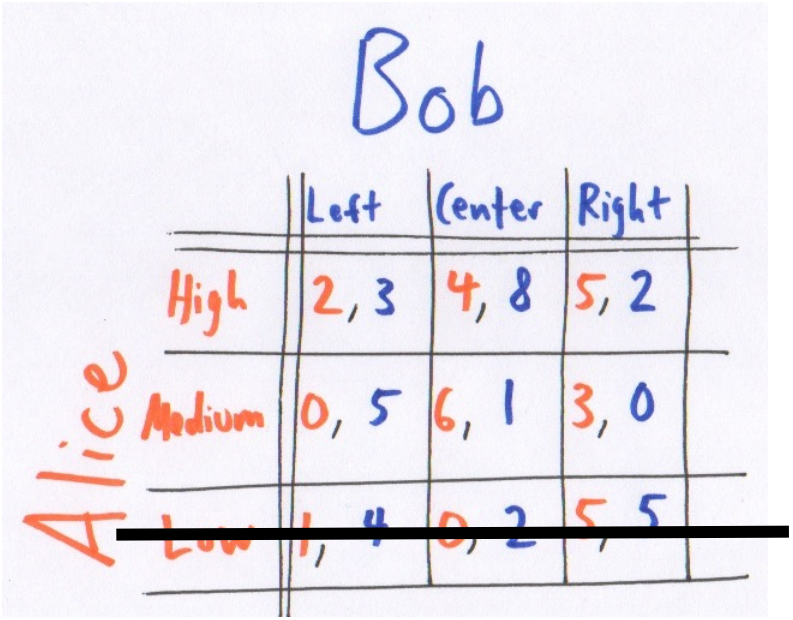

- For Alice, Low is ***dominated*** by High
- Now for Bob, Right is dominated by Center

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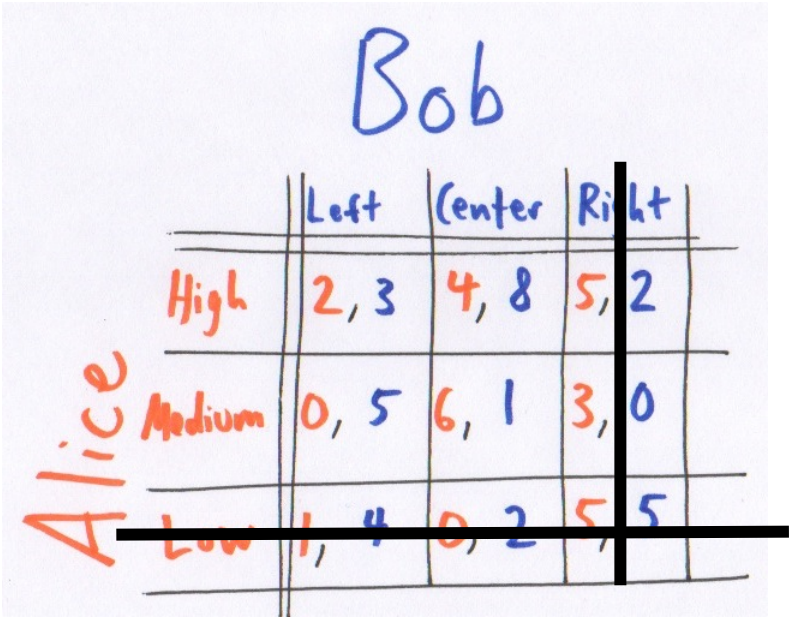

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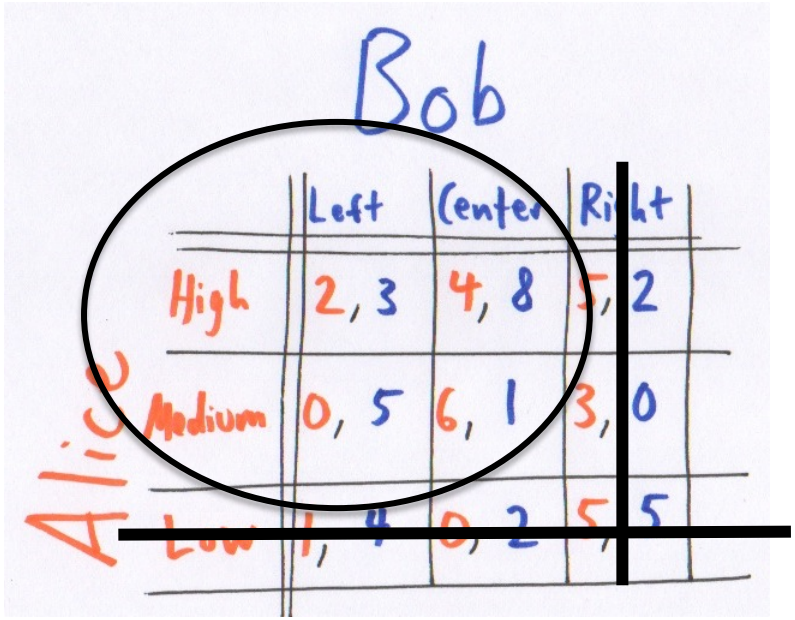

- For Alice, Low is ***dominated*** by High
- Now for Bob, Right is dominated by Center
- So the only ***rationalizable*** outcomes are in {High, Medium} x {Left, Center}

Bob

Alice

	$100	$99	$98	$97	...	$3	$2
$100	100, 100	97, 101	96, 100	95, 99	...	1, 5	0, 4
$99	101, 97	99, 99	96, 100	95, 99	...	1, 5	0, 4
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Dominance and Traveler's Dilemma

- Only "rationalizable" outcome is ($2, $2)!

Dominance and Traveler's Dilemma

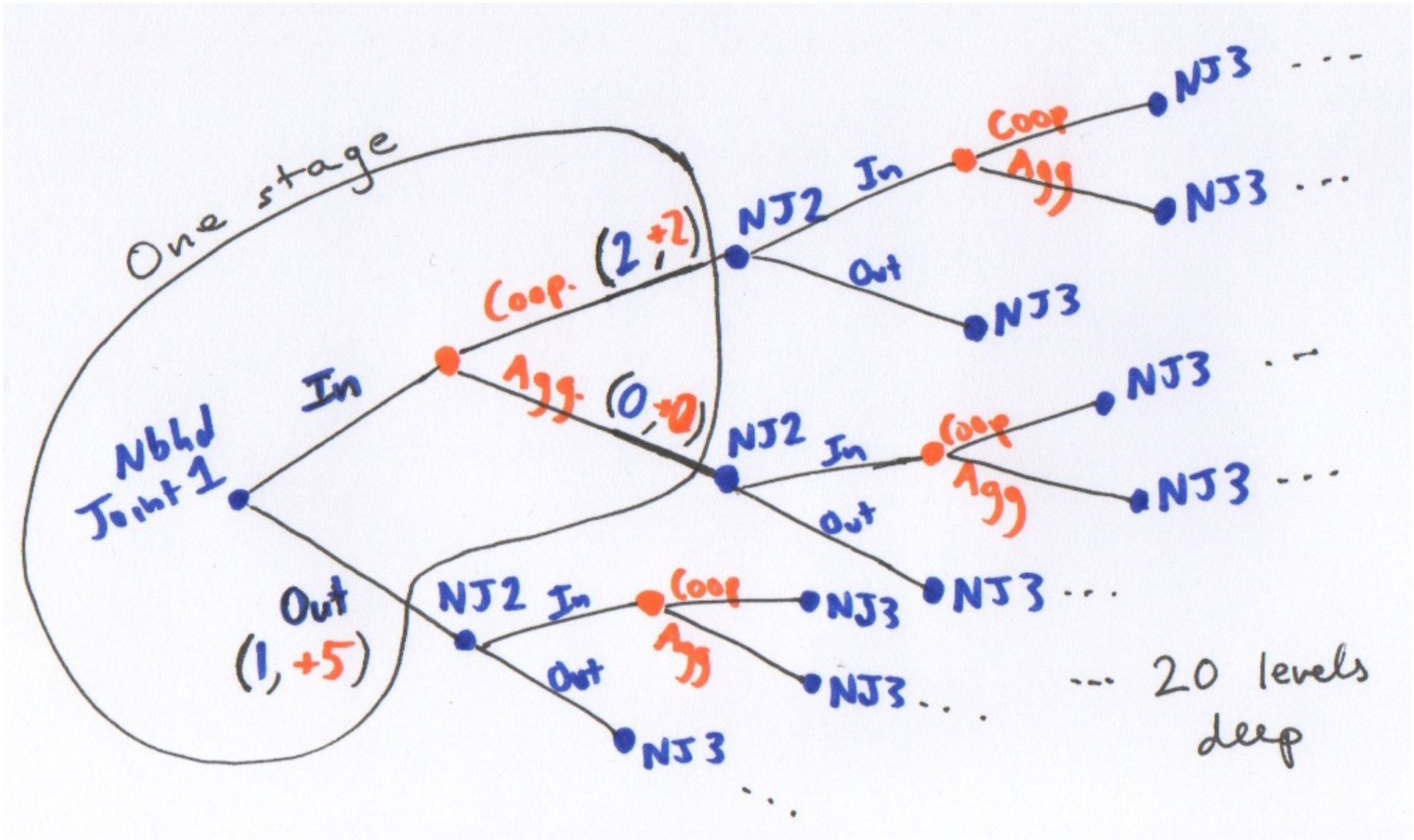
- Only "rationalizable" outcome is ($2, $2)!
- wtf?
- Surely we have a better solution concept than this…

Chain Store Paradox (Reinhard Selten, 1978)

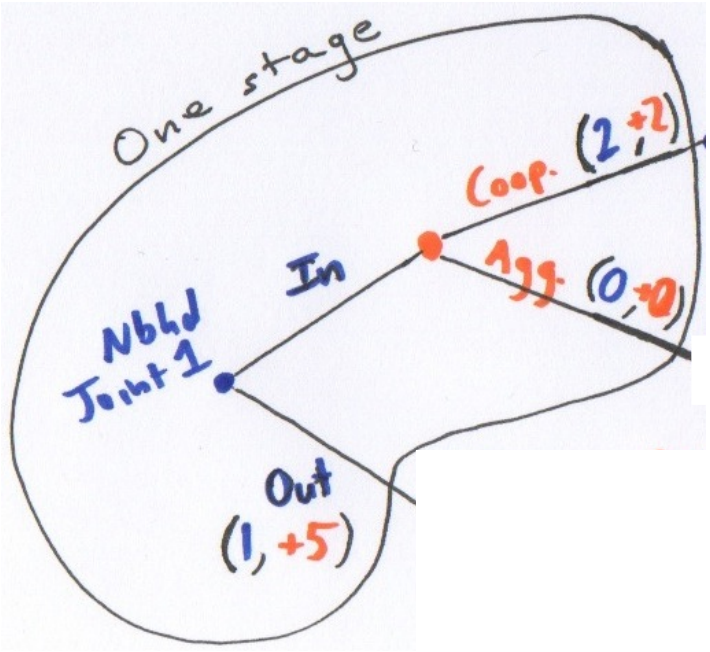
vs.

20 neighborhood burger joints, one by one

- Neighborhood joints: enter market or not?
- If so, McDonalds cooperative or aggressive?

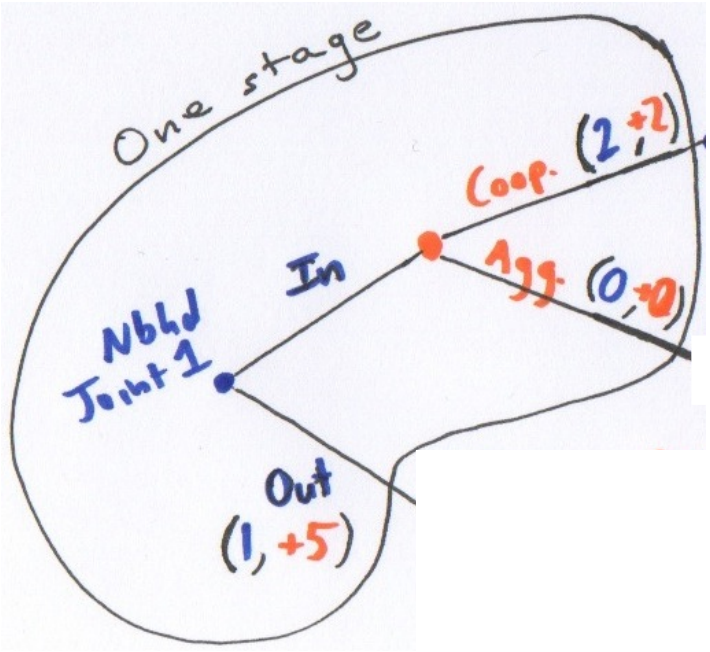

One stage
Nbhd Joint 1
In
Out
(1, +5)
Coop.
(2, +2)
Agg.
(0, +0)
NJ2
In
Out
Coop
Agg
NJ3
... 20 levels deep

Backwards Induction



- At stage 20, McD's should always Coop if NJ20 is In

Backwards Induction



- At stage 20, McD's should always Coop if NJ20 is In
- Thus NJ20 should get In

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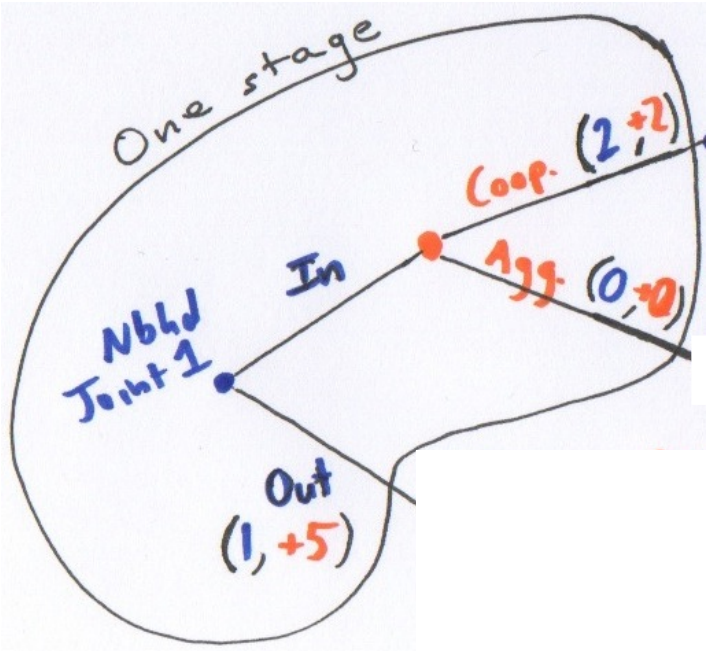
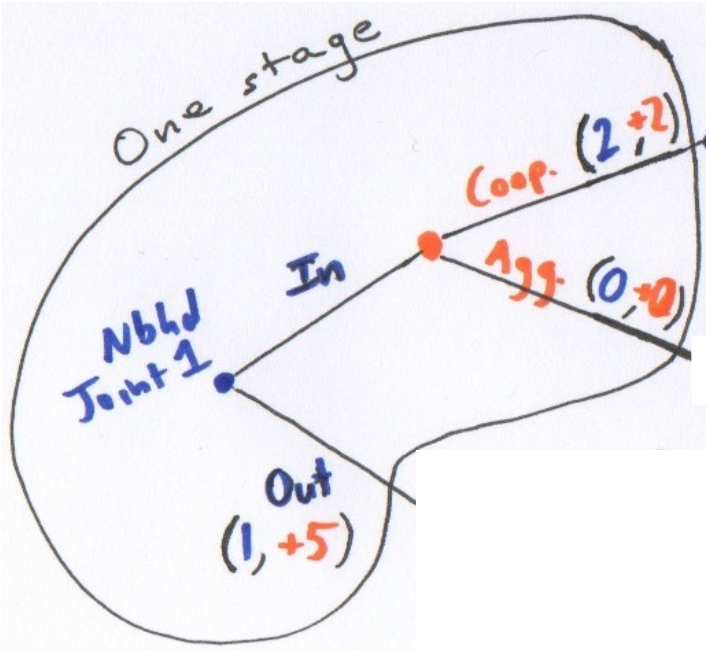


- At stage 20, McD's should always Coop if NJ20 is In
- Thus NJ20 should get In
- So stage 20 is determined. Thus at stage 19… etc.

Backwards Induction



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- Thus NJ20 should get In
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- This yields 40 for McD's and 2 for each nbhd joint

But Wait!

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- What if McDonald's is aggressive in the first round?

Selten's Explanation

- Three "levels" of decision-making:
  - "routine" (past experience)
  - "imagination" (how will my decision affect future?)
  - "reasoning" ("rational" analysis, i.e. game theory)

Another possible direction: *n*th order rationality

- Keynesian Beauty Contest

- Guess 2/3 of the average guess

Punch line?

And why am I not an economist?