FORM GR9367

67

GRE®

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MATHEMATICS TEST

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MATHEMATICS TEST

Time—170 minutes

66 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. In each case, select the one that is the best of the choices offered and then mark the corresponding space on the answer sheet.

Computation and scratchwork may be done in this examination book.

Note: In this examination:

(1) All logarithms are to the base e unless otherwise specified.
(2) The set of all x such that $a \le x \le b$ is denoted by $[a, b]$.

1. If $f(g(x)) = 5$ and $f(x) = x + 3$ for all real x, then $g(x) =$

(A) $x - 3$ (B) $3 - x$ (C) $\frac{5}{x+3}$ (D) 2 (E) 8

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2. $\lim_{x \to 0} \frac{\tan x}{\cos x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) $+\infty$

---

3. $\int_0^{\log 4} e^{2x} dx =$

(A) $\frac{15}{2}$ (B) 8 (C) $\frac{17}{2}$ (D) $\frac{\log 16}{2} - 1$ (E) $\log 4 - \frac{1}{2}$

---

4. Let $A - B$ denote $\{x \in A : x \notin B\}$. If $(A - B) \cup B = A$, which of the following must be true?

(A) B is empty
(B) $A \subseteq B$
(C) $B \subseteq A$
(D) $(B - A) \cup A = B$
(E) None of the above

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5. If $f(x) = |x| + 3x^2$ for all real x, then $f'(-1)$ is

(A) -7 (B) -5 (C) 5 (D) 7 (E) nonexistent

6. For what value of b is the value of $\int_b^{b+1} (x^2 + x)\,dx$ a minimum?

(A) 0 (B) -1 (C) -2 (D) -3 (E) -4

7. In how many of the eight standard octants of xyz-space does the graph of $z = e^{x+y}$ appear?

(A) One (B) Two (C) Three (D) Four (E) Eight

8. Suppose that the function f is defined on an interval by the formula $f(x) = \sqrt{\tan^2 x - 1}$. If f is continuous, which of the following intervals could be its domain?

(A) $\left(\frac{3\pi}{4}, \pi\right)$

(B) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

(C) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

(D) $\left(-\frac{\pi}{4}, 0\right)$

(E) $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$

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9. $\int_0^1 \frac{x}{2 - x^2}\, dx =$

(A) $-\frac{1}{2}$ (B) $\frac{5}{3}$ (C) $\frac{\log 2 - e}{2}$ (D) $-\frac{\log 2}{2}$ (E) $\frac{\log 2}{2}$

---

10. If $f''(x) = f'(x)$ for all real x, and if $f(0) = 0$ and $f'(0) = -1$, then $f(x) =$

(A) $1 - e^x$ (B) $e^x - 1$ (C) $e^{-x} - 1$ (D) e^{-x} (E) $-e^x$

---

11. If $\phi(x, y, z) = x^2 + 2xy + xz^{\frac{3}{2}}$, which of the following partial derivatives are identically zero?

I. $\frac{\partial^2 \phi}{\partial y^2}$

II. $\frac{\partial^2 \phi}{\partial x \partial y}$

III. $\frac{\partial^2 \phi}{\partial z \partial y}$

(A) III only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II, and III

GO ON TO THE NEXT PAGE.

12. $\lim_{x \to 0} \dfrac{\sin 2x}{(1 + x)\log(1 + x)} =$

(A) -2 (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

---

13. $\lim_{n \to \infty} \int_1^n \frac{1}{x^n}\,dx =$

(A) 0 (B) 1 (C) e (D) π (E) $+\infty$

---

14. At a 15 percent annual inflation rate, the value of the dollar would decrease by approximately one-half every 5 years. At this inflation rate, in approximately how many years would the dollar be worth $\frac{1}{1{,}000{,}000}$ of its present value?

(A) 25 (B) 50 (C) 75 (D) 100 (E) 125

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15. Let $f(x) = \int_1^x \frac{1}{1+t^2}\,dt$ for all real x. An equation of the line tangent to the graph of f at the point $(2, f(2))$ is

(A) $y - 1 = \frac{1}{5}(x - 2)$ (B) $y - \text{Arctan } 2 = \frac{1}{5}(x - 2)$ (C) $y - 1 = (\text{Arctan } 2)(x - 2)$

(D) $y - \text{Arctan } 2 + \frac{\pi}{4} = \frac{1}{5}(x - 2)$ (E) $y - \frac{\pi}{2} = (\text{Arctan } 2)(x - 2)$

---

16. Let $f(x) = e^{g(x)}h(x)$ and $h'(x) = -g'(x)h(x)$ for all real x. Which of the following must be true?

(A) f is a constant function.
(B) f is a linear nonconstant function.
(C) g is a constant function.
(D) g is a linear nonconstant function.
(E) None of the above

---

17. $1 - \sin^2\left(\text{Arccos } \frac{\pi}{12}\right) =$

(A) $\sqrt{\dfrac{1 - \cos\frac{\pi}{24}}{2}}$ (B) $\sqrt{\dfrac{1 - \cos\frac{\pi}{6}}{2}}$ (C) $\sqrt{\dfrac{1 + \cos\frac{\pi}{24}}{2}}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi^2}{144}$

GO ON TO THE NEXT PAGE.

18. If $f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$ for all $x \in (0, 1)$, then $f'(x) =$

(A) $\sin x$ (B) $\cos x$ (C) $\frac{1}{1+x^2}$ (D) $\frac{-2x}{(1+x^2)^2}$ (E) $\frac{2x}{(1-2x)^2}$

---

19. Which of the following is the general solution of the differential equation

$$\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = 0\ ?$$

(A) $c_1e^t + c_2te^t + c_3t^2e^t$

(B) $c_1e^{-t} + c_2te^{-t} + c_3t^2e^{-t}$

(C) $c_1e^t - c_2e^{-t} + c_3te^{t^2}$

(D) $c_1e^t + c_2e^{2t} + c_3e^{3t}$

(E) $c_1e^{2t} + c_2te^{-2t}$

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20. Which of the following double integrals represents the volume of the solid bounded above by the graph of $z = 6 - x^2 - 2y^2$ and bounded below by the graph of $z = -2 + x^2 + 2y^2$?

(A) $4\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(B) $\int_{x=-2}^{x=2} \int_{y=-\sqrt{(4-x^2)/2}}^{y=\sqrt{(4-x^2)/2}} (8 - 2x^2 - 4y^2)\, dy\, dx$

(C) $4\int_{y=0}^{y=\sqrt{2}} \int_{x=-\sqrt{4-2y^2}}^{x=\sqrt{4-2y^2}} dx\, dy$

(D) $\int_{y=-\sqrt{2}}^{y=\sqrt{2}} \int_{x=-2}^{x=2} (8 - 2x^2 - 4y^2)\, dx\, dy$

(E) $2\int_{y=0}^{y=\sqrt{2}} \int_{x=0}^{x=\sqrt{4-2y^2}} (8 - 2x^2 - 4y^2)\, dx\, dy$

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21. Let a be a number in the interval $[0, 1]$ and let f be a function defined on $[0, 1]$ by

$$f(x) = \begin{cases} a^2 & \text{if } 0 \le x \le a, \\ ax & \text{otherwise.} \end{cases}$$

Then the value of a for which $\int_0^1 f(x)\, dx = 1$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) nonexistent

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22. If b and c are elements in a group G, and if $b^5 = c^3 = e$, where e is the unit element of G, then the inverse of $b^2cb^4c^2$ must be

(A) b^3c^2bc (B) $b^4c^2b^2c$ (C) $c^2b^4cb^2$ (D) $cb^2c^2b^4$ (E) cbc^2b^3

---

23. Let f be a real-valued function continuous on the closed interval $[0, 1]$ and differentiable on the open interval $(0, 1)$ with $f(0) = 1$ and $f(1) = 0$. Which of the following must be true?

I. There exists $x \in (0, 1)$ such that $f(x) = x$.
II. There exists $x \in (0, 1)$ such that $f'(x) = -1$.
III. $f(x) > 0$ for all $x \in [0, 1)$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

---

24. If A and B are events in a probability space such that $0 < P(A) = P(B) = P(A \cap B) < 1$, which of the following CANNOT be true?

(A) A and B are independent.
(B) A is a proper subset of B.
(C) $A \neq B$
(D) $A \cap B = A \cup B$
(E) $P(A)P(B) < P(A \cap B)$

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25. Let f be a real-valued function with domain [0, 1]. If there is some $K > 0$ such that $f(x) - f(y) \leq K|x - y|$ for all x and y in [0, 1], which of the following must be true?

(A) f is discontinuous at each point of (0, 1).
(B) f is not continuous on (0, 1), but is discontinuous at only countably many points of (0, 1).
(C) f is continuous on (0, 1), but is differentiable at only countably many points of (0, 1).
(D) f is continuous on (0, 1), but may not be differentiable on (0, 1).
(E) f is differentiable on (0, 1).

---

26. Let $\mathbf{i} = (1, 0, 0)$, $\mathbf{j} = (0, 1, 0)$, and $\mathbf{k} = (0, 0, 1)$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal if $\mathbf{v}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{v}_2 =$

(A) $\mathbf{i} + \mathbf{j} - \mathbf{k}$ (B) $\mathbf{i} - \mathbf{j} + \mathbf{k}$ (C) $\mathbf{i} + \mathbf{k}$ (D) $\mathbf{j} - \mathbf{k}$ (E) $\mathbf{i} + \mathbf{j}$

---

27. If the curve in the yz-plane with equation $z = f(y)$ is rotated around the y-axis, an equation of the resulting surface of revolution is

(A) $x^2 + z^2 = [f(y)]^2$
(B) $x^2 + z^2 = f(y)$
(C) $x^2 + z^2 = |f(y)|$
(D) $y^2 + z^2 = |f(y)|$
(E) $y^2 + z^2 = [f(x)]^2$

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28. Let A and B be subspaces of a vector space V. Which of the following must be subspaces of V?

I. $A + B = \{a + b:\ a \in A \text{ and } b \in B\}$
II. $A \cup B$
III. $A \cap B$
IV. $\{x \in V:\ x \notin A\}$

(A) I and II only
(B) I and III only
(C) III and IV only
(D) I, II, and III only
(E) I, II, III, and IV

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29. $\lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{2^k}\right) =$

(A) 0 (B) 1 (C) 2 (D) 4 (E) $+\infty$

---

30. If $f(x_1, \ldots, x_n) = \sum_{1 \le i < j \le n} x_i x_j$, then $\frac{\partial f}{\partial x_n} =$

(A) $n!$ (B) $\sum_{1 \le i < j < n} x_i x_j$ (C) $\sum_{1 \le i < j < n} (x_i + x_j)$ (D) $\sum_{j=1}^{n} x_j$ (E) $\sum_{j=1}^{n-1} x_j$

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31. If $f(x) = \begin{cases} \sqrt{1 - x^2} & \text{for } 0 \le x \le 1 \\ x - 1 & \text{for } 1 < x \le 2, \end{cases}$

then $\int_0^2 f(x)\,dx$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{1}{2} + \frac{\pi}{4}$

(D) $\frac{1}{2} + \frac{\pi}{2}$

(E) undefined

---

32. Let R denote the field of real numbers, Q the field of rational numbers, and Z the ring of integers. Which of the following subsets F_i of R, $1 \le i \le 4$, are subfields of R?

$$F_1 = \{a/b\colon\ a, b \in Z \text{ and } b \text{ is odd}\}$$
$$F_2 = \{a + b\sqrt{2}\colon\ a, b \in Z\}$$
$$F_3 = \{a + b\sqrt{2}\colon\ a, b \in Q\}$$
$$F_4 = \{a + b\sqrt[4]{2}\colon\ a, b \in Q\}$$

(A) No F_i is a subfield of R.
(B) F_3 only
(C) F_2 and F_3 only
(D) F_1, F_2, and F_3 only
(E) F_1, F_2, F_3, and F_4

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33. If n apples, no two of the same weight, are lined up at random on a table, what is the probability that they are lined up in order of increasing weight from left to right?

(A) $\frac{1}{2}$ (B) $\frac{1}{n}$ (C) $\frac{1}{n!}$ (D) $\frac{1}{2^n}$ (E) $\left(\frac{1}{n}\right)^n$

---

34. $\frac{d}{dx}\int_0^{x^2} e^{-t^2}\,dt =$

(A) e^{-x^2} (B) $2e^{-x^2}$ (C) $2e^{-x^4}$ (D) $x^2e^{-x^2}$ (E) $2xe^{-x^4}$

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35. Let f be a real-valued function defined on the set of integers and satisfying $f(x) = \frac{1}{2}f(x-1) + \frac{1}{2}f(x+1)$. Which of the following must be true?

I. The graph of f is a subset of a line.
II. f is strictly increasing.
III. f is a constant function.

(A) None
(B) I only
(C) II only
(D) I and II
(E) I and III

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36. If F is a function such that, for all positive integers x and y, $F(x, 1) = x + 1$, $F(1, y) = 2y$, and $F(x+1, y+1) = F(F(x, y+1), y)$, then $F(2, 2) =$

(A) 8 (B) 7 (C) 6 (D) 5 (E) 4

---

37. If $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} = 9$, then $\det\begin{pmatrix} 3a & 3b & 3c \\ g-4a & h-4b & k-4c \\ d & e & f \end{pmatrix} =$

(A) -108 (B) -27 (C) 3 (D) 12 (E) 27

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38. $\lim_{n \to \infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(\frac{3i}{n} \right)^2 - \left(\frac{3i}{n} \right) \right] =$

(A) $-\frac{1}{6}$ (B) 0 (C) 3 (D) $\frac{9}{2}$ (E) $\frac{31}{6}$

---

39. For a real number x, $\log(1 + \sin 2\pi x)$ is not a real number if and only if x is

(A) an integer

(B) nonpositive

(C) equal to $\frac{2n-1}{2}$ for some integer n

(D) equal to $\frac{4n-1}{4}$ for some integer n

(E) any real number

---

40. If x, y, and z are selected independently and at random from the interval [0, 1], then the probability that $x \geq yz$ is

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$ (E) $\frac{1}{4}$

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41. If $A = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$, then the set of all vectors X for which $AX = X$ is

(A) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \ a = 0 \text{ and } b \text{ is arbitrary} \right\}$

(B) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \ a \text{ is arbitrary and } b = 0 \right\}$

(C) $\left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| \ a = -b \text{ and } b \text{ is arbitrary} \right\}$

(D) $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

(E) the empty set

---

42. What is the greatest value of b for which any real-valued function f that satisfies the following properties must also satisfy $f(1) < 5$?

(i) f is infinitely differentiable on the real numbers;
(ii) $f(0) = 1$, $f'(0) = 1$, and $f''(0) = 2$; and
(iii) $|f'''(x)| < b$ for all x in $[0, 1]$.

(A) 1 (B) 2 (C) 6 (D) 12 (E) 24

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43. Let n be an integer greater than 1. Which of the following conditions guarantee that the equation

$$x^n = \sum_{i=0}^{n-1} a_i x^i$$ has at least one root in the interval (0, 1) ?

I. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i < 1$

II. $a_0 > 0$ and $\sum_{i=0}^{n-1} a_i > 1$

III. $a_0 < 0$ and $\sum_{i=0}^{n-1} a_i > 1$

(A) None
(B) I only
(C) II only
(D) III only
(E) I and III

---

44. If x is a real number and P is a polynomial function, then $\lim_{h \to 0} \dfrac{P(x+3h) + P(x-3h) - 2P(x)}{h^2} =$

(A) 0 (B) $6P'(x)$ (C) $3P''(x)$ (D) $9P''(x)$ (E) ∞

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45. Consider the system of equations

$$ax^2 + by^3 = c$$
$$dx^2 + ey^3 = f$$

where a, b, c, d, e, and f are real constants and $ae \neq bd$. The maximum possible number of real solutions (x, y) of the system is

(A) none (B) one (C) two (D) three (E) five

46. If $x^3 - x + 1 = a_0 + a_1(x - 2) + a_2(x - 2)^2 + a_3(x - 2)^3$ for all real numbers x, then (a_0, a_1, a_2, a_3) is

(A) $\left(1, \frac{1}{2}, 0, -\frac{1}{8}\right)$

(B) $(1, -1, 0, 1)$

(C) $(7, 6, 10, 1)$

(D) $(7, 11, 12, 6)$

(E) $(7, 11, 6, 1)$

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47. Let C be the ellipse with center $(0, 0)$, major axis of length $2a$, and minor axis of length $2b$. The value of $\oint_C x\,dy - y\,dx$ is

(A) $\pi\sqrt{a^2 + b^2}$

(B) $2\pi\sqrt{a^2 + b^2}$

(C) $2\pi ab$

(D) πab

(E) $\frac{\pi ab}{2}$

---

48. Let G_n denote the cyclic group of order n. Which of the following direct products is NOT cyclic?

(A) $G_{17} \times G_{11}$
(B) $G_{17} \times G_{11} \times G_5$
(C) $G_{17} \times G_{33}$
(D) $G_{22} \times G_{33}$
(E) $G_{49} \times G_{121}$

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49. Let X be a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{4}(1 - x^2) & \text{if } -1 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the standard deviation of X ?

(A) 0 (B) $\frac{1}{5}$ (C) $\frac{\sqrt{30}}{15}$ (D) $\frac{1}{\sqrt{5}}$ (E) 1

---

50. The set of all points (x, y, z) in Euclidean 3-space such that

$$\begin{vmatrix} 1 & x & y & z \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{vmatrix} = 0$$

is

(A) a plane containing the points (1, 0, 0), (0, 1, 0), and (0, 0, 1)
(B) a sphere with center at the origin and radius 1
(C) a surface containing the point (1, 1, 1)
(D) a vector space with basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)}
(E) none of the above

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51. An automorphism ϕ of a field F is a one-to-one mapping of F onto itself such that $\phi(a+b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in F$. If F is the field of rational numbers, then the number of distinct automorphisms of F is

(A) 0 (B) 1 (C) 2 (D) 4 (E) infinite

52. Let T be the transformation of the xy-plane that reflects each vector through the x-axis and then doubles the vector's length.

If A is the 2×2 matrix such that $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \end{bmatrix}$ for each vector $\begin{bmatrix} x \\ y \end{bmatrix}$, then $A =$

(A) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} \frac{\sqrt{2}}{2} & 1 \\ 1 & \frac{-\sqrt{2}}{2} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$

(E) $\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

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53. Let $r > 0$ and let C be the circle $|z| = r$ in the complex plane. If P is a polynomial function, then $\int_C P(z)\,dz =$

(A) 0
(B) πr^2
(C) $2\pi i$
(D) $2\pi P(0)i$
(E) $P(r)$

---

54 If f and g are real-valued differentiable functions and if $f'(x) \geq g'(x)$ for all x in the closed interval [0, 1], which of the following must be true?

(A) $f(0) \geq g(0)$

(B) $f(1) \geq g(1)$

(C) $f(1) - g(1) \geq f(0) - g(0)$

(D) $f - g$ has no maximum on [0, 1]

(E) $\frac{f}{g}$ is a nondecreasing function on [0, 1].

---

55. Let p and q be distinct primes. There is a proper subgroup J of the additive group of integers which contains exactly three elements of the set $\{p, p + q, pq, p^q, q^p\}$. Which three elements are in J?

(A) pq, p^q, q^p
(B) $p + q, pq, p^q$
(C) $p, p + q, pq$
(D) p, p^q, q^p
(E) p, pq, p^q

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56. For a subset S of a topological space X, let $\mathrm{cl}(S)$ denote the closure of S in X, and let $S' = \{x : x \in \mathrm{cl}(S - \{x\})\}$ denote the derived set of S. If A and B are subsets of X, which of the following statements are true?

I. $(A \cup B)' = A' \cup B'$
II. $(A \cap B)' = A' \cap B'$
III. If A' is empty, then A is closed in X.
IV If A is open in X, then A' is not empty.

(A) I and II only
(B) I and III only
(C) II and IV only
(D) I, II, and III only
(E) I, II, III, and IV

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57. Consider the following procedure for determining whether a given name appears in an alphabetized list of n names.

Step 1. Choose the name at the middle of the list (if $n = 2k$, choose the kth name); if that is the given name, you are done; if the list is only one name long, you are done. If you are not done, go to Step 2.

Step 2. If the given name comes alphabetically before the name at the middle of the list, apply Step 1 to the first half of the list; otherwise, apply Step 1 to the second half of the list.

If n is very large, the maximum number of steps required by this procedure is close to

(A) n
(B) n^2
(C) $\log_2 n$
(D) $n \log_2 n$
(E) $n^2 \log_2 n$

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58. Which of the following is an eigenvalue of the matrix

$$\begin{pmatrix} 2 & 1-i \\ 1+i & -2 \end{pmatrix}$$

over the complex numbers?

(A) 0 (B) 1 (C) $\sqrt{6}$ (D) i (E) $1 + i$

---

59. Two subgroups H and K of a group G have orders 12 and 30, respectively. Which of the following could NOT be the order of the subgroup of G generated by H and K?

(A) 30 (B) 60 (C) 120 (D) 360 (E) Countable infinity

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60. Let A and B be subsets of a set M and let $S_0 = \{A, B\}$. For $i \geq 0$, define S_{i+1} inductively to be the collection of subsets X of M that are of the form $C \cup D$, $C \cap D$, or $M - C$ (the complement of C in M), where $C, D \in S_i$. Let $S = \bigcup_{i=0}^{\infty} S_i$. What is the largest possible number of elements of S?

(A) 4
(B) 8
(C) 15
(D) 16
(E) S may be infinite.

GO ON TO THE NEXT PAGE.

61. A city has square city blocks formed by a grid of north-south and east-west streets. One automobile route from City Hall to the main firehouse is to go exactly 5 blocks east and 7 blocks north. How many different routes from City Hall to the main firehouse traverse exactly 12 city blocks?

(A) $5 \cdot 7$

(B) $\frac{7!}{5!}$

(C) $\frac{12!}{7!5!}$

(D) 2^{12}

(E) $7!5!$

---

62. Let R be the set of real numbers with the topology generated by the basis $\{[a, b): a < b, \text{ where } a, b \in R\}$. If X is the subset $[0, 1]$ of R, which of the following must be true?

I. X is compact.
II. X is Hausdorff.
III. X is connected.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) II and III

GO ON TO THE NEXT PAGE.

63. Let R be the circular region of the xy-plane with center at the origin and radius 2. Then $\iint_R e^{-(x^2+y^2)}dx\,dy =$

(A) 4π

(B) πe^{-4}

(C) $4\pi e^{-4}$

(D) $\pi(1 - e^{-4})$

(E) $4\pi(e - e^{-4})$

64. Let V be the real vector space of real-valued functions defined on the real numbers and having derivatives of all orders. If D is the mapping from V into V that maps every function in V to its derivative, what are all the eigenvectors of D?

(A) All nonzero functions in V

(B) All nonzero constant functions in V

(C) All nonzero functions of the form $ke^{\lambda x}$, where k and λ are real numbers

(D) All nonzero functions of the form $\sum_{i=0}^{k} c_i x^i$, where $k > 0$ and the c_i's are real numbers

(E) There are no eigenvectors of D.

GO ON TO THE NEXT PAGE.

65. If f is a function defined by a complex power series expansion in $z - a$ which converges for $|z - a| < 1$ and diverges for $|z - a| > 1$, which of the following must be true?

(A) $f(z)$ is analytic in the open unit disk with center at a

(B) The power series for $f(z + a)$ converges for $|z + a| < 1$.

(C) $f'(a) = 0$

(D) $\int_C f(z)dz = 0$ for any circle C in the plane.

(E) $f(z)$ has a pole of order 1 at $z = a$.

---

66. Let n be any positive integer and $1 \le x_1 < x_2 < \ldots < x_{n+1} \le 2n$, where each x_i is an integer. Which of the following must be true?

I. There is an x_i that is the square of an integer.
II. There is an i such that $x_{i+1} = x_i + 1$.
III. There is an x_i that is prime.

(A) I only
(B) II only
(C) I and II
(D) I and III
(E) II and III

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST.

to eliminate one or more of the answer choices, your chance of getting the right answer is improved, and it may be to your advantage to answer such a question.

- Record all answers on your answer sheet. Answers recorded in your test book will not be counted.
- Do not wait until the last five minutes of a testing session to record answers on your answer sheet.

HOW TO SCORE YOUR TEST (GR9367)

Total Subject Test scores are reported as three-digit scaled scores with the third digit always zero. The maximum possible range for all Subject Test total scores is from 200 to 990. The actual range of scores for a particular Subject Test, however, may be smaller. The possible range for GRE Mathematics Test scores is from 400 to 990. The range for different editions of a given test may vary because different editions are not of precisely the same difficulty. The differences in ranges among different editions of a given test, however, usually are small. This should be taken into account, especially when comparing two very high scores. In general, differences between scores at the 99th percentile should be ignored. **The score conversion table provided shows the score range for this edition of the test only.**

The work sheet on page 25 lists the correct answers to the questions. Columns are provided for you to mark whether you chose the correct (C) answer or an incorrect (I) answer to each question. Draw a line across any question you omitted, because it is not counted in the scoring. At the bottom of the page, enter the total number correct and the total number incorrect. Divide the total incorrect by 4 and subtract the resulting number from the total correct. This is the adjustment made for guessing. Then round the result to the nearest whole number. This will give you your raw total score. Use the total score conversion table to find the scaled total score that corresponds to your raw total score.

Example: Suppose you chose the correct answers to 48 questions and incorrect answers to 15. Dividing 15 by 4 yields 3.75. Subtracting 3.75 from 48 equals 44.25, which is rounded to 44. The raw score of 44 corresponds to a scaled score of 870.

WORK SHEET for the GRE Mathematics Test, Form GR9367
Answer Key and Percentages* of Examinees Answering Each Question Correctly

QUESTION Number	Answer	P+	TOTAL C	I
1	D	96		
2	C	95		
3	A	84		
4	C	76		
5	A	70		
6	B	86		
7	D	80		
8	B	76		
9	E	74		
10	A	94		
11	C	82		
12	E	69		
13	A	65		
14	D	62		
15	D	56		
16	A	63		
17	E	69		
18	D	48		
19	A	57		
20	B	52		
21	D	79		
22	E	73		
23	C	61		
24	A	51		
25	D	33		
26	C	69		
27	A	59		
28	B	45		
29	E	68		
30	E	56		
31	C	64		
32	B	36		
33	C	88		
34	E	60		
35	B	45		

QUESTION Number	Answer	P+	TOTAL C	I
36	D	70		
37	B	66		
38	D	23		
39	D	69		
40	A	40		
41	B	75		
42	D	31		
43	E	17		
44	D	31		
45	C	53		
46	E	83		
47	C	18		
48	D	58		
49	D	28		
50	A	38		
51	B	37		
52	D	72		
53	A	49		
54	C	57		
55	E	28		
56	B	17		
57	C	72		
58	C	65		
59	A	49		
60	D	22		
61	C	52		
62	B	10		
63	D	49		
64	C	50		
65	A	60		
66	B	53		

Correct (C) ____________

Incorrect (I) ____________

Total Score:

C – I/4 = ____________

Scaled Score (SS) = ____________

*The P+ column indicates the percent of GRE Mathematics Test examinees who answered each question correctly; it is based on a sample of February 1993 examinees selected to represent all GRE Mathematics Test examinees tested between October 1, 1996 and September 30, 1999

Score Conversions and Percents Below* for GRE Mathematics Test, Form GR9367

TOTAL SCORE					
Raw Score	Scaled Score	%	Raw Score	Scaled Score	%
55-66	990	82	26	690	33
54	980	81	25	680	31
53	970	80	24	670	30
52	960	79	23	660	28
51	950	77	22	650	26
50	930	74	21	640	24
49	920	72	20	630	23
48	910	71	19	620	22
47	900	69	18	600	18
46	890	67	17	590	17
45	880	66	16	580	15
44	870	64	15	570	13
43	860	62	14	560	12
42	850	61	13	550	11
41	840	59	12	540	10
40	830	57	11	530	8
39	820	55	10	520	7
38	810	54	9	510	6
37	800	52	8	500	5
36	790	51	7	490	4
35	780	49	6	480	2
34	770	47	5	470	2
33	760	45	4	460	1
32	750	43	3	450	1
31	740	41	2	440	0
30	730	39	1	430	0
29	720	38	0	420	0
28	710	37			
27	700	35			

*Percent scoring below the scaled score is based on the performance of 7,092 examinees who took the GRE Mathematics Test between October 1, 1996 and September 30, 1999. This percent below information was used for score reports during the 2000-01 testing year.

EVALUATING YOUR PERFORMANCE (GR9367)

Now that you have scored your test, you may wish to compare your performance with the performance of others who took this test. A worksheet and table are provided, both using performance data from GRE Mathematics Test examinees.

The worksheet (on page 25) is based on the performance of a sample of the examinees who took this particular test in February1993 This sample was selected to represent the total population of GRE Mathematics Test examinees tested between October 1996 and September 1999. On the work sheet you used to determine your score is a column labeled "P+." The numbers in this column indicate the percent of the examinees in this sample who answered each question correctly. You may use these numbers as a guide for evaluating your performance on each test question.

The table on page 26 contains, for each scaled score, the percentage of examinees tested between October 1996 and September 1999 who received lower scores. Interpretive data based on the scores earned by examinees tested in this three-year period were used by admissions officers in 2000-2001. These percentages appear in the score conversion table in a column to the right of the scaled scores. For example, in the percent column opposite the scaled score of 870 is the number 64. This means that 64 percent of the GRE Mathematics Test examinees tested between October 1996 and September 1999 scored lower than 870. To compare yourself with this population, look at the percent next to the scaled score you earned on the practice test. This number is a reasonable indication of your rank among GRE Mathematics Test examinees if you followed the test-taking suggestions in this practice book.

It is important to realize that the conditions under which you tested yourself were not exactly the same as those you will encounter at a test center. It is impossible to predict how different test-taking conditions will affect test performance, and this is only one factor that may account for differences between your practice test scores and your actual test scores. By comparing your performance on this practice test with the performance of other GRE Mathematics Test examinees, however, you will be able to determine your strengths and weaknesses and can then plan a program of study to prepare yourself for taking the GRE Mathematics Test under standard conditions.

Before you start timing yourself on the test that follows, we suggest that you remove an answer sheet (pages 131 to 136) and turn first to the back cover of the test book (page 130), as you will do at the test center, and follow the instructions for completing the identification areas of the answer sheet. When you are ready to begin the test, note the time and start marking your answers to the questions on the answer sheet.