

Problem statement Suppose $f(x) = x^{-\ln x}$.

a) Verify that $\lim_{x \rightarrow 0^+} f(x) = 0$ and $\lim_{x \rightarrow \infty} f(x) = 0$. Graph f on the interval $[0, 10]$.

b) A remarkable result of third semester calculus is that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$. Assume that this result is correct, and use it to show that $\int_0^{\infty} x^{-\ln x} dx = e^{1/4} \sqrt{\pi}$. (Maple can “do” the first integral, but not the second!)

Hint Make a change of variables, and then complete the square.

c) Include a graph of e^{-x^2} on the interval $[-2, 2]$. Use your answer to b) to compare this graph to the graph in a).