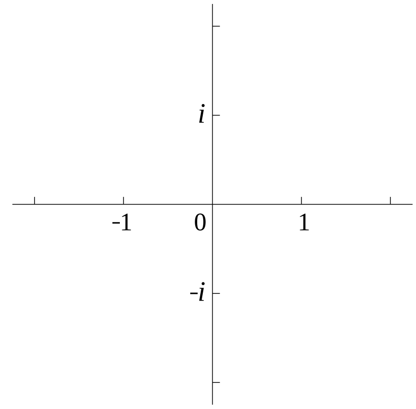
(10) 1. Describe all solutions of $z^3 = -8i$ algebraically in rectangular form. Sketch the solutions on the axes provided.



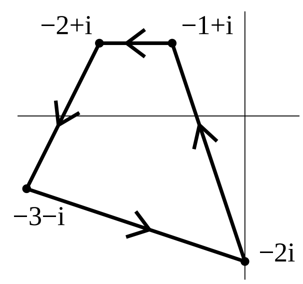
(10) 2. Verify that if t is real, then $\dfrac{1}{t+i}$ is on the circle of radius $\dfrac{1}{2}$ centered at $-\dfrac{i}{2}$.

Hint Compute directly the distance from the point to the given center and verify that it is the given radius.

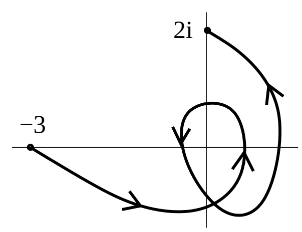
(10) 3. Suppose f is an entire function and that $\mathrm{Re}\big(f(z)\big) = \mathrm{Im}\big(f(z)\big)$ for all $z \in \mathbb{C}$. Show that f is constant.

Hint The Cauchy-Riemann equations.

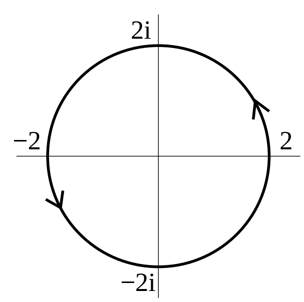
(10) 4. Compute $\displaystyle\int_C e^{(z^2+5z-i)}dz$ where C is the indicated closed curve.



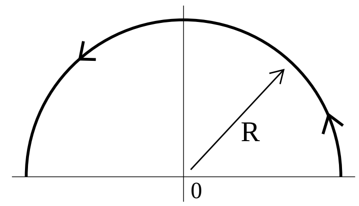
(10) 5. Compute $\displaystyle\int_C z^2 - \frac{1}{z^2}\, dz$ where C is the indicated curve.



(10) 6. Compute $\displaystyle\int_C z^3 - \frac{1}{z}\, dz$ where C is the indicated closed curve (a circle oriented counterclockwise centered at 0 with radius 2).



(10) 7. Suppose S_R is the upper semicircle of radius $R > 0$ centered at the origin as shown, A, B, and C are <u>positive</u> numbers, and $f(z) = \dfrac{e^{iAz}}{z^2 + Bz + C}$. Prove that $\lim\limits_{R\to\infty} \displaystyle\int_{S_R} f(z)\, dz = 0$.



Hint *ML*.

(10) 8. Consider the sum $\displaystyle\sum_{j=1}^{\infty} \frac{z^j}{3^j j^2}$.

a) Find the radius of convergence of this series.

b) For which z's does this series converge absolutely? For which z's must the series diverge? (Please *omit* behavior on the boundary of the circle of convergence.)

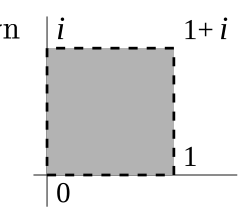
c) For which z's does the series converge? (Again, please *omit* behavior on the boundary of the circle of convergence.) Briefly explain why the answer to b) implies an answer to this question.

(10) 9. a) Find all real numbers A and B so that the function $x^4 + Ax^2y^2 + By^4$ is harmonic.

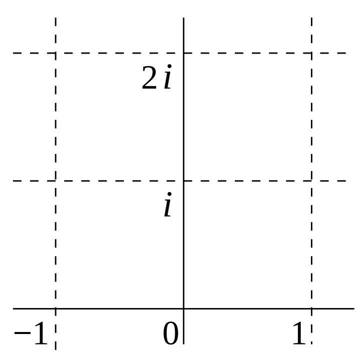
b) If $f(x, y)$ and $g(x, y)$ are both harmonic, must the sum $f(x, y) + g(x, y)$ always be harmonic also? If yes, briefly give a reason. If no, give an example so that $f(x, y) + g(x, y)$ is *not* harmonic.

c) If $f(x, y)$ and $g(x, y)$ are both harmonic, must the product $f(x, y)g(x, y)$ always be harmonic also? If yes, briefly give a reason. If no, give an example so that $f(x, y)g(x, y)$ is *not* harmonic.

(10) 10. Suppose S is the open square with corners at 0, 1, $1 + i$, and i, shown to the right.

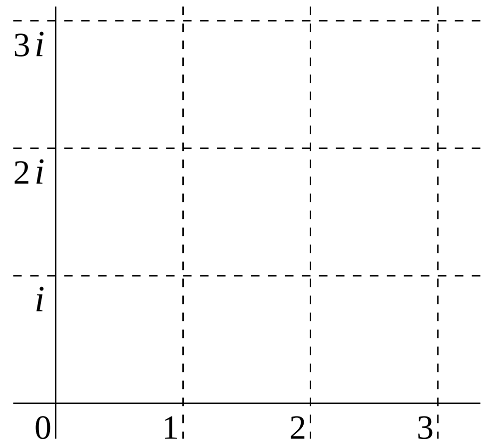


a) Sketch the image of S on the axes to the right under the mapping $z \to z^2$. But sure to indicate as well as you can what happens to the corners, the sides, and the interior of the square.



b) Sketch the image of S (the original square!) on the axes below under the mapping $z \to \exp(z) = e^z$. But sure to indicate as well as you can what happens to the corners, the sides, and the interior of the square.

Comment Please realize that $\dfrac{\pi}{4} < 1 < \dfrac{\pi}{2}$. You *don't need* exact values to make a sketch!



First Exam for Math 403, section 1

March 10, 2008

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes, texts, or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total Points Earned:		