Math 403:01 **Entrance "exam"** **January 20, 2008**

Due at the beginning of class, Monday, January 25, 2010

1. (4) Compute $\displaystyle\int_1^2 \frac{dx}{1-2x}$ and simplify.

2. (12) Suppose f is a differentiable function of one variable, $f'(x) = g(x)$, and $f(0) = A$. Suppose that g is also a differentiable function of one variable, $g'(x) = \big(f(x)\big)^2$, and $g(0) = B$. Now define $F(x,y) = f(x^2 - y^3)$. Compute $F(1,1)$, $\dfrac{\partial F}{\partial x}(1,1)$, $\dfrac{\partial F}{\partial y}(1,1)$, and $\dfrac{\partial^3 F}{\partial x \partial^2 y}(1,1)$ in terms of A and B.

3. (8) Find *all* functions $v(x,y)$ such that both $\dfrac{\partial v}{\partial x} = x^2 + y$ and $\dfrac{\partial v}{\partial y} = x - y^3$ are correct.

4. (6) What shape (square, line, disk, circle, etc.) is described by the collection of ordered pairs $\{(2 + 3\cos\theta, 3\sin\theta)\}$ when $0 \le \theta \le \pi$? Sketch this shape.

5. (10) Suppose $f(x) = \dfrac{\cos(2x)}{1+x}$.
a) Find A, B, C, D, and E so that $f(x) = A + Bx + Cx^2 + Dx^3 + Ex^4$ with an error less than $|x|^5$ for x near 0. (This is the initial segment of the Taylor series for f centered at 0.)
b) Use your answer to a) to compute $f^{(4)}(0)$.

6. (12) Suppose A is a positive real number and that I_A is defined by (A appears three times in the formula!) by $I_A = \displaystyle\int_A^{2A} e^{-(Ax^2)}\,dx$.
What is the sign of the integrand? What are the maximum and minimum values of the integrand in the interval of integration and why? How long is the interval of integration? Use this information to show that for any positive integer, n, $\displaystyle\lim_{A\to\infty} A^n I_A = 0$.

7. (8) Compute the line integral of $x^2\,dx + xy\,dy$ along two paths: the straight line from $(0,0)$ to $(1,2)$, and the parabolic arc $y = 2x^2$ from $(0,0)$ to $(1,2)$.

Rules Please treat this as any other homework assignment. That is, you may consult textbooks or acquaintances or me (!), but the written work you hand in must be your own. <u>An answer alone will not receive full credit</u> – you must show supporting computation or give some explanation or both. I will grade what you hand in as an exam. A passing grade will be at least 75% of the 60 points. Familiarity with all of the material tested here is necessary for success in this course.

Comments In problem 2, solutions can be found in terms of the Weierstrass $\wp$ (pronounced "P") function, important in complex analysis. But please *don't* solve the equations! In problem 5, *don't* try to compute the values directly but use known facts about the pieces of the function. In problem 6, no one can find an antiderivative in terms of familiar functions, so *don't* try to use the Fundamental Theorem of Calculus. Instead, estimate the integral. Most of the problems can be solved with standard calculus methods so **read your calculus book!**