(15) 1. This problem is about the following system of linear equations:

$$\begin{cases} 3x_1 - 2x_2 + 2x_3 + 1x_4 = a \\ -6x_1 + 4x_2 + 4x_3 + 5x_4 = b \\ -15x_1 + 10x_2 + 6x_3 + 9x_4 = c \\ 18x_1 - 12x_2 - 4x_3 - 8x_4 = d \\ -9x_1 + 6x_2 + 2x_3 + 4x_4 = e \end{cases}$$

After some row operations, the augmented matrix of this system becomes what follows.

$$\begin{bmatrix} 1 & -\frac{2}{3} & 0 & -\frac{1}{4} & -\frac{1}{12}b + \frac{1}{6}a \\ 0 & 0 & 1 & \frac{7}{8} & \frac{1}{8}b + \frac{1}{4}a \\ 0 & 0 & 0 & 0 & a - 2b + c \\ 0 & 0 & 0 & 0 & -2a + 2b + d \\ 0 & 0 & 0 & 0 & -b + a + e \end{bmatrix}$$

The following questions refer to the *coefficient matrix* of the system of linear equations. In this problem, you need not supply verification of your answers.
a) Give the dimension and a basis of the <u>null space</u> of the coefficient matrix.
b) Give the dimension and a basis of the <u>column space</u> of the coefficient matrix.
c) Give the dimension and a basis of the <u>row space</u> of the coefficient matrix.

(10) 2. a) Suppose that S is a set of vectors in $\mathbb{R}^n$. Define "*S is a subspace of* $\mathbb{R}^n$".
b) Suppose S is the set of vectors in $\mathbb{R}^2$ given by $\begin{bmatrix} x \\ y \end{bmatrix}$ where $y = x^2$ and x is any real number. Use your answer to part a) to explain why S is <u>not</u> a subspace of $\mathbb{R}^2$.

(15) 3. In this problem A is the 2×2 matrix $\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$. The aim is to *diagonalize* A. That is, we want to write $A = PDP^{-1}$ where D is a diagonal matrix and P is an invertible matrix.
a) Find the characteristic polynomial of A and find the two distinct eigenvalues of A.
b) Find an eigenvector for each of the eigenvalues listed in a).
c) Use the results of a) and b) to write D and P.
d) Find P^{-1}.
e) Verify PDP^{-1} is A: **first** compute PD; **then** multiply the result on the right by P^{-1}.

(6) 4. Suppose S is a subspace of $\mathbb{R}^{500}$ which contains a set of 4 linearly independent vectors and which is spanned by 6 of its vectors. What are the possible values of the dimension of S? You *must* give reasoning which supports your answer to earn full credit.

(12) 5. a) Suppose $A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is a 3×3 matrix with row vectors a, b, and c. Assume that $\det A = 5$. Find the determinant of the matrix $\begin{bmatrix} c + b \\ a + 2b \\ a - b - c \end{bmatrix}$.

b) Compute the determinant of the matrix $\begin{bmatrix} 3 & 4 & 0 & 0 \\ 1 & -1 & 1 & 2 \\ 2 & 0 & 2 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$.

(12) 6. **True or false** (An answer alone will not receive full credit!)
a) If A is a 10×13 matrix, then the nullspace of A is *not* $\{0\}$.
b) A basis of $\mathbb{R}^4$ is given by the vectors $\left\{ \begin{bmatrix} 1 \\ -3 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 8 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 3 \\ -2 \end{bmatrix} \right\}$.
c) If A and B are any 2×2 matrices, then the matrix products AB and BA are equal.

(12) 7. Suppose A is the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ and B is the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.
For which values of μ is $A - \mu B$ invertible? (Copy A and B carefully!)

(18) 8. In this problem A is a 4×4 matrix, and v is an eigenvector of A with corresponding eigenvalue 3. You *must* give reasoning which supports your answer to earn full credit.
a) Is $-2v$ an eigenvector of A? If it is, what is its corresponding eigenvalue?
b) Is v an eigenvector of $5A$? If it is, what is its corresponding eigenvalue?
c) Is v an eigenvector of A^2? If it is, what is its corresponding eigenvalue?

Second Exam for Math 250, sections 1 & 5

April 13, 2011

Name _____ **Section** (please circle one) 1 5

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes and no calculators may be used on this exam.
"Simplification" of answers is not necessary.

Problem Number	Possible Points	Points Earned:
1	15	
2	10	
3	15	
4	6	
5	12	
6	12	
7	12	
8	18	
Total Points Earned:		