Answers to

Quiz #8A for Math 250:1 & 5

4/6/2011

Name ______________________ **Section** (please circle one) 1 5

In these problems, A is the 4×4 matrix $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

1. (8) a) Find the characteristic polynomial of A and the eigenvalues of A.

Answer The determinant of $A-\lambda I_4$ is $(2-\lambda)(-1-\lambda)^2(-\lambda)$ (the matrix is upper-triangular so the determinant is the product of the diagonal elements). The eigenvalues are 0, -1, and 2.

b) Find a basis for each eigenspace of A.

Answer $\underline{\lambda = 0}$. Then $A - \lambda I_4$ is $\begin{bmatrix} 2 & 1 & 1 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. The third row implies that $x_3 = 0$. Then the second row implies that $x_2 = 0$ and the first row implies that $x_1 = 0$. A basis for this eigenspace is $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right]$.

$\underline{\lambda = -1}$. Then $A - \lambda I_4$ is $\begin{bmatrix} 3 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. The last row shows that $x_4 = 0$ and the second row shows that $x_3 = 0$. The first row implies that $x_2 = -3x_1$. A basis for this eigenspace is $\left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \\ 0 \end{bmatrix} \right\}$.

$\underline{\lambda = 2}$. Then $A - \lambda I_4$ is $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & -3 & 2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$. The last row shows that $x_4 = 0$, the third row shows that $x_3 = 0$, and the second row then implies the x_2 must be 0. The variable x_1 is not controlled: it is *free*. A basis for this eigenspace is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

2. (2) Can A be diagonalized? Briefly support your assertion with some reasoning.

Answer <u>No</u> A has a maximum number of *three* linearly independent eigenvectors in $\mathbb{R}^4$. We need *four* for diagonalization.