1. For each of the four cases below, sketch a graph of a function that satisfies the stated conditions. In each case, the *domain* of the function should be *all real numbers*.

a) $\lim_{x\to 2} f(x) = 3$ and $f(2) = 4$.

b) $\lim_{x\to 0} f(x)$ does not exist, and $|f(x)| < 2$ for all x.

c) $\lim_{x\to 1} f(x)$ exists and its value is $f(1) + 2$.

d) $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1^+} f(x)$ do not exist, $|f(x)| < 3$ for all x, and $f(-1) = -2$.

2. Suppose $f(x)$ is a piecewise function defined as follows:

$$f(x) = \begin{cases} 2x^2 + 2, & \text{if } x < 1 \\ ax^2 + bx, & \text{if } 1 \le x \le 2 \\ 2 - \dfrac{6}{x}, & \text{if } x > 2 \end{cases}.$$

a) Suppose that $a = 2$ and $b = -3$. Graph $f(x)$ for $0 \le x \le 3$. Find the left and right hand limits of $f(x)$ as x approaches 1 and as x approaches 2.

b) Find a and b so that the graph of $f(x)$ doesn't have any jumps (that is, $f(x)$ is continuous everywhere). Graph the resulting function $f(x)$ for $0 \le x \le 3$.

3. a) Suppose that $f(x) = x^2$ and $g(x) = 2^x$. Compute $f(-2)$, $g(-2)$, $f(5)$, and $g(5)$. According to the Intermediate Value Theorem and the function values computed, what is the smallest number of roots the equation $f(x) = g(x)$ can have?

b) Suppose still that $f(x) = x^2$ and $g(x) = 2^x$. Graph $y = f(x)$ and $y = g(x)$ carefully on the interval $-2 \le x \le 5$. How many roots does the equation $f(x) = g(x)$ appear to have?

c) Draw graphs of two increasing continuous functions which intersect exactly two times.

d) Draw graphs of two increasing continuous functions which intersect exactly three times.

e) Draw graphs of two increasing continuous functions which intersect exactly four times.

4. Some lines which are tangent to the parabola $y = x^2$ also pass through the point $(2, 3)$. Find all of these lines. Graph the parabola and the tangent lines which were found on the same coordinate axes.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 153 webpage to learn which problem to hand in.