Problems for 153

1. Evaluate each of these limits numerically to two decimal places or state that the limit does not exist. (Please read the formulas carefully!)

a)  $\lim_{x \to 1} x^{\left(\frac{1}{x-1}\right)}$ . b)  $\lim_{x \to 2} \frac{x^x - 4}{x^2 - 4}$ .

This is taken from the textbook's Chapter Review Exercises for Chapter 2.

2. Suppose that f is the function defined by the formula  $f(x) = \left(\arctan\left(\ln(\sqrt{x}-1)\right)\right)^3$ .

a) What are the domain and range of f? Answers should *not* be numerical approximations, but should be written if needed in terms of traditional constants such as  $\pi$  and e.

b) If y = f(x), write a formula for x in terms of y.

3. a) Suppose  $f_n(x) = x^n e^{-x}$ . Graph  $f_n(x)$  for  $0 \le x \le 10$  and n = 1, 2, 3. You may need to adjust the viewing window to see the graph. Describe how the graphs change as nincreases. What features stay the same? Find the x coordinate  $x_{\max}$  of the highest point of the graph for n = 1, 2, 3. Plot  $x_{\max}$  as a function of n. Guess what the graph of  $f_5(x)$ looks like, and what the x coordinate of the highest point is. Then test your guess by actually generating the graph.

b) Suppose  $g_n(x) = xe^{-nx}$ . Graph  $g_n(x)$  for  $0 \le x \le 10$  and n = 1, 2, 3. You may need to adjust the viewing window to see the graph. Describe how the graphs change as n increases. What features stay the same? Find the x coordinate  $x_{\max}$  of the highest point of the graph for n = 1, 2, 3. Plot  $x_{\max}$  as a function of n. Guess what the graph of  $g_5(x)$  looks like, and what the x coordinate of the highest point is. Then test your guess by actually generating the graph.

4. The Dubois & Dubois formula\* for body surface area is  $BSA = \frac{\text{weight}^{0.425} \cdot \text{height}^{0.725}}{139.2}$ 

Here BSA, body surface area, is measured in square meters, weight is measured in kilograms, and height is measured in centimeters. BSA is very important in many medical and other "human factors" applications (cooling, for example). Appropriate units with explanations should accompany all of your answers in this problem.

a) What is the BSA for a person<sup>\*\*</sup> whose weight is 83 kilograms and whose height is 186.7 centimeters?

b) If the person mentioned in a) gained 1 kilogram (not too difficult for him), what would be the rate of change of that person's BSA?

c) If the person mentioned in a) magically grew 1 centimeter taller, what would be the rate of change of that person's BSA?

d) Compute the change in the BSA for each of the situations in b) and c). Which change is greater?

<sup>\*</sup> This is a real formula, used widely since it was first published in 1916.

<sup>\*\*</sup> A random calculus lecturer  $\underline{is}$  a person.