Math 151, Fall 2009, Review Problems for Exam 2

Your second exam is likely to have problems that do not resemble these review problems. Partial answers to these problems will be posted in a few days.

(1) Find $\displaystyle\lim_{x\to 1} \frac{2x^4 - 3x^3 + x^2 - x + 1}{x^4 - 3x^3 + 2x^2 + x - 1}$.

(2) Find $\displaystyle\lim_{x\to 0} \frac{x - \sin x}{x - x\cos x}$.

(3) Find the horizontal asymptotes of $f(x) = \dfrac{x}{\sqrt{7x^2+1}}$.

(4) For each function given below, find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima, the inflection points, the horizontal asymptotes and the vertical asymptotes.

(a) $f(x) = \dfrac{x^2}{x^2-4}$ (b) $g(x) = \dfrac{x}{x^2-4}$.

(5) For the function $f(x) = x^5 - 3x^3 + 4x$, find the intervals where it is increasing, the intervals where it is decreasing, the intervals where it is concave up, the intervals where it is concave down, the local maxima, the local minima and the inflection points.

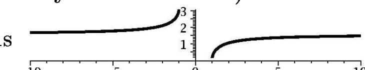
(6) For the function $f(x) = \dfrac{1}{\sqrt{x^2+1}}$, find the intervals where it is concave up, the intervals where it is concave down and the inflection points.

(7) Find all local maxima and all local minima of $f(x) = \cos x - \dfrac{\sin x}{\sqrt{3}}$.

(8) Let $g(x)$ be the inverse of $f(x) = x^3 + 2x + 4$. Calculate $g(7)$ [without finding a formula for $g(x)$] and then calculate $g'(7)$.

This is a textbook problem (3.9, exercise 9) given as a substitute for the coordinator's problem concerning arcsec. I firmly believe that arcsec is too bizarre for this course!

(9) Find the absolute maximum and the absolute minimum of $f(x) = \ln(-x) + \sec^{-1} x$ over the interval $[-2, -1]$.

O.k., look up the derivative of arcsec on the formula sheet (I certainly don't know it!) in order to complete this problem. I *still* believe that a function whose graph looks like this 

is too strange.

(10) Find $\dfrac{d}{dx}\left[(\ln x)^x\right]$ for $x > 1$.

(11) Find the slope of the tangent to the curve $x^2y^3 + xy = 78$ at the point $(3, 2)$ on the curve.

(12) Find the linearization of $f(x) = x^{1/3}$ with center $a = 27$.

(13) A spherical weather balloon is being inflated at the rate of 12 cubic inches per second. What is the radius of the balloon when its surface area is increasing at a rate of 5 square inches per second?

(14) Find the area between the x-axis, the line $x = 1$ and the parabola $y = x^2$ in the following way: Approximate the area using the sum of the areas of n rectangles. Let n approach infinity.

Please skip this because the topic will not be tested on our exam.

(15) Find $\sqrt{7}$ with an accuracy of 0.000001 using Newton's Method.

I don't know how to do this without a calculator which is not "legal" on our exams. Please use a calculator only for this practice problem.

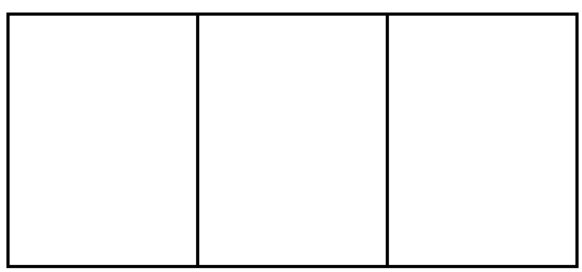
(16) Find all functions $f(x)$ such that $f''(x) = x(x^2 - 1)$.

Please skip this because the topic will not be tested on our exam.

(17) Consider a rectangular industrial warehouse consisting of three separate spaces of equal size as in the figure shown. Assume that the wall materials cost \$200 per linear ft and the company allocates \$2,400,000 for the project*.

(a) What dimensions maximize the total area of the warehouse?

(b) What is the area of each compartment in this case?



The course coordinator has come up with a wonderful and very elaborate problem which I believe is unrealistic for an exam but which is quite imaginative. I prefer to have you "rehearse" with something more conventional, more resembling what you might get on an exam. This is section 4.6's exercise 24.

* Presumably there are no other costs.