Questions for 151 Televised Final Review

1) Find

$$\lim_{x \to 0} x^x$$

2) Find $f'(x)$ where

$$f(x) = (\cos x)^x$$

3) Approximate $(8.1)^{1/3}$ using linearization.

4) Find L_4 and R_4 for $f(x) = x^2$ on the interval $[1, 3]$.

5) Two circles have the same center. The inner circle has radius r which is increasing at $3in/s$. The outer circle has radius R which is increasing at $2in/s$. Suppose that A is the area of the region between the circles. When $r = 7in$ and $R = 10in$, what is A? How fast is A changing? Is A increasing or decreasing?

6) Find $\dfrac{dy}{dx}$ if $y^3 = 3xy + y^2 + 4x^3$.

7) Find $\dfrac{dy}{dx}$ if $y = (\ln(x^2) + 2x^3)^{1/3}$.

8) Calculate

$$\lim_{x \to 0} \frac{\sin x - x}{e^x - 1}$$

9) Calculate the following limits:

$$\lim_{x \to 3} \frac{x^2 + 5x + 6}{x^2 + 2x - 3}, \quad \lim_{x \to -3} \frac{x^2 + 5x + 6}{x^2 + 2x - 3}, \quad \lim_{x \to \infty} \frac{x^2 + 5x + 6}{x^2 + 2x - 3}$$

10) Let $f(x) = \cos(3x) + 3x^2 - 6x + 1$. Explain why there must be a root of $f(x) = 0$ in $[-1, 1]$.

11) Suppose that the function f is continuous on $[1, 3]$ and differentiable on $(1, 3)$. If $f(1) = 9$ and $f'(x) \geq 2$ for $1 \leq x \leq 3$, how small can $f(3)$ possibly be?

12) Find the value of c that makes the function continuous.

$$f(x) = \begin{cases} x^2 - c & \text{for } x < 5 \\ 4x + 2c & \text{for } x \geq 5 \end{cases}$$

13) Describe the set $S = \{x \in \mathbb{R} : |4x - 3| > 2 \text{ and } |x - 2| \leq 1\}$ in terms of intervals.

14) Complete the square for $4x^2 - 6x + 1$ to solve $4x^2 - 6x + 1 = 0$.

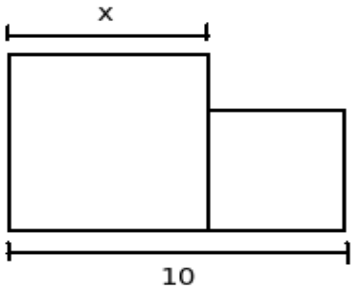
15) Find the horizontal and vertical asymptotes of

$$\frac{2x}{\sqrt{3x^2 - 1}}$$

16) Find the equation of the tangent line to the function $f(x) = x^3 + 5x + 2$ at $a = 3$.

17) Two squares are placed so their sides are touching as shown. The sum of the lengths of one side of each square is $10ft$. Suppose the length of the left square is x feet. The left square is painted with paint costing \$6 per square foot.

The right square is painted with paint costing \$4 per square foot. For which x will the cost be a minimum?



18) Simplify $\cos(\sin^{-1}(x^2))$.

19) Solve $\ln(x^2+10) - \ln(x^2+1) = 3\ln 2$

20) Use the limit definition of the derivative to calculate the derivative of $f(t) = t^{-2}$ at $a = 1$.

21) Find all functions $f(x)$ such that $f''(x) = x^2(x^3+2x+1)$.

22) Differentiate $\tan^{-1}(\ln x)$, $\ln(\sin^{-1}(x^2))$, $(x^2+3)/(2x^2+1)$ and $\sin^3(e^{2x}+\ln x)$.

23) Let $f(x) = (1-x^2)^4$. Find the intervals where $f(x)$ is increasing, decreasing, concave up, and concave down. Give any inflection points and local extremas.

24) Find the maximum and minimum values of $f(x) = (\cos x)(\sin x)$ on the interval $[0, \frac{\pi}{2}]$.

25) Let $f(x) = \dfrac{1}{x+1}$. Give the intervals on which f is increasing, decreasing, concave up, concave down. Give any asymptotes, inflection points and local extremas. Use this information to sketch the graph of $f(x)$.

26) Use two iterations of Newton's method to approximate $\sqrt{5}$.

27) Suppose $f(x)$ is a differentiable function with $f(9) = 3$, $f'(9) = 6$ and $f''(9) = -2$. If $F(x) = f(x^2)$, compute $F(3)$, $F'(3)$ and $F''(3)$.

28) Find constants a, b such that the function $f(x)$, defined below, is differentiable.

$$f(x) = \begin{cases} ax+1 & \text{if } x < 1 \\ x^2+b & \text{if } x \geq 1 \end{cases}$$

29) Evaluate the integrals: $\int (9t-4)^{11}dt$, $\int_1^2 \dfrac{4t}{t^2+1}dt$.

30) Calculate

$$\lim_{x\to 0} \frac{\int_0^x e^{t^2}}{x}$$

31) Assume that $f(t)$ is a function such that $\int_1^4 f(t)dt = -2$, $\int_0^5 f(t)dt = 3$ and $\int_4^5 f(t)dt = -1$. Find $\int_0^1 f(t)dt$.