1. Graph the functions $f(x) = \frac{4x}{x^2+1}$ and $g(x) = 2\sin(2\arctan x)$ in the same viewing window over the interval $-5 \le x \le 5$. Explain what you see.

2. Suppose that $f(x) = e^{-Ax}$, where A is a positive real number.

a) Show that the integral $\int_1^2 f(x)\,dx \to 0$ as $A \to \infty$. (You may wish to draw a picture, but other verification is also necessary.)

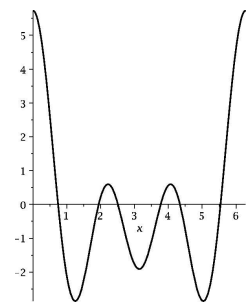
b) Show that the integral $\int_1^2 xf(x)\,dx \to 0$ as $A \to \infty$. (You may wish to draw a picture, but other verification is also necessary.)

c) Show that the integral $\int_1^2 \frac{x^{17}}{1+5x^{48}} f(x)\,dx \to 0$ as $A \to \infty$. (You may wish to draw a picture, but other verification is also necessary.)

Note It isn't always necessary or even possible to compute every integral exactly. But this integral can be *estimated* to get enough information.

3. a) Compute $\int_0^{2\pi} \big(\cos(mx)\big)\big(\cos(nx)\big)\,dx$ if m and n are integers.
(Be careful: there are two different results, one when $m = n$ and one when $m \ne n$.)

b) If $f(x) = A\cos(x) + B\cos(2x) + C\cos(3x)$, $\int_0^{2\pi} f(x)\cos(x)\,dx = 5$, $\int_0^{2\pi} f(x)\cos(2x)\,dx = 6$, and $\int_0^{2\pi} f(x)\cos(3x)\,dx = 7$, then find A and B and C.



Note The ideas of this computation are used often with Fourier series, a standard method of analyzing periodic phenomena. A graph of f is shown to the right. Your ear (and some mechanical and electrical devices) can find 5 and 6 and 7 in this graph!

4. a) Suppose A is a positive real number and m_A is the average value of $\big(\sin(Ax)\big)^3$ on the interval $[0, 2]$. Compute m_A.

Note The answer will have several terms and will *not* be simple.

b) What is $\lim\limits_{A\to\infty} m_A$?

Note This answer *should* be simple. Explain briefly why it is correct. You may refer to graphs of functions if that is helpful.

5. An oil tank has the shape of a cylinder whose diameter is 4 feet. It is mounted so that the axis of the cylinder is horizontal (the circular cross-sections of the cylinder are vertical). If the depth of the water is 3 feet, what percentage of the total capacity of the tank is filled?

After drawing a picture and setting up this problem, solve it three ways:

a) Use elementary geometry (compare areas of circular sectors).

b) Express the answer in terms of a definite integral, then obtain an approximate numerical value for the integral using the `fnInt(` function on your calculator.

c) Evaluate the integral in b) exactly in terms of elementary functions using a trig substitution, then obtain approximate numerical values for these functions using your calculator.